

## 27408 - Mathematics II

### Syllabus Information

**Academic year:** 2023/24

**Subject:** 27408 - Mathematics II

**Faculty / School:** 109 - Facultad de Economía y Empresa

**Degree:** 417 - Degree in Economics

**ECTS:** 6.0

**Year:** 1

**Semester:** Second semester

**Subject type:** Basic Education

**Module:**

### 1. General information

Mathematics in the Economics degree will support other key subjects such as Microeconomics, Macroeconomics and Econometrics.

Mathematics instruction in this year has two main goals: to train students in mathematics and to train them to use Mathematics in their future profession. In addition to the goals covered in Mathematics I, the aim is to develop a rigorous approach, abstraction capacity and the scientific method characteristic of Mathematics. Modelling techniques related to classical optimization, linear programming and dynamic analysis will be also introduced.

These approaches are aligned with the Sustainable Development Goals (SDGs) of the UN's 2030 Agenda, as the mathematical modelling can be applied to all 17 goals.

It is recommended to have taken Mathematics I.

### 2. Learning results

At the end of the subject the students will be able to:

- 1.- Acquire skills in the use of mathematical language, both in comprehension and writing.
- 2.- Identify the fundamental elements of an optimization problem: variables, objective function and constraints.
- 3.- Pose static optimization problems with and without equality and inequality constraints.
- 4.- Solve optimization problems graphically, when possible.
- 5.- Evaluate whether a mathematical program meets the conditions to be solved by means of the techniques studied.
- 6.- Distinguish between critical and extreme or optimal points.
- 7.- Distinguish between local optima and global optima.
- 8.- Distinguish between necessary and sufficient conditions of local optimality.
- 9.- Calculate the critical points by solving the system of equations obtained by posing the first order conditions of local optimality, both in unconstrained and with equality constraints.
- 10.- Study the critical points using the second order conditions, in problems without constraints and with equality constraints.
- 11.- Apply conditions that ensure the globality of the optimums.
- 12.- Interpret the economic significance of Lagrange multipliers in optimization problems with equality constraints.
- 13.- Evaluate whether a mathematical program is linear and solve it graphically, if possible, and by means of the simplex algorithm.
- 14.- Analyse the variation in the solution of a linear optimization problem in response to changes in the data, without solving a new problem.
- 15.- Identify a dynamic process in an economic phenomenon and represent it by means of an ordinary differential equation, if possible.
- 16.- Understand the concept of solution of an ordinary differential equation and distinguish between general solution and particular solution.
- 17.- Distinguish between first order differential equation and linear differential equation of order  $n$ .
- 18.- Solve some first order differential equations using the appropriate method.
- 19.- Distinguish in a linear differential equation of constant coefficients the associated homogeneous equation and calculate its general solution
- 20.- Calculate a particular solution of a linear differential equation of constant coefficients.
- 21.- Calculate the general solution of a linear differential equation of constant coefficients.

- 22.- Calculate the solution of a linear differential equation of constant coefficients of order  $n$  with  $n$  initial conditions.
- 23.- Use qualitative analysis in simple dynamic models in an economic context to identify equilibrium and its long-run scope.
- 24.- Identify the fundamental elements in an economic problem, formalize it as a mathematical problem, solve it with the most appropriate tool and interpret the results in the original economic context.

### 3. Syllabus

#### Unit 1: Mathematical programs

- 1.1. General formulation of a mathematical program. Classification.
- 1.2. Definitions and properties. Weierstrass theorem.
- 1.3. Graphic resolution.
- 1.4- Introduction to convexity:
  - 1.4.1. Convex sets. Definition and properties.
  - 1.4.2. Convex and concave functions. Definitions and properties.
  - 1.4.3. Convex programs.

#### Unit 2: Unrestricted programming

- 2.1- Problem formulation.
- 2.2- Optimal premises:
  - 2.2.1. First-order conditions for the existence of local optimality.
  - 2.2.2. Second-order conditions for the existence of local optimality.
- 2.3- Global optimums: Convex programs.

#### Unit 3: Programming with equality constraints

- 3.1- Problem formulation.
- 3.2- Optimal premises:
  - 3.2.1. First-order conditions for the existence of local optimality.
  - 3.2.2. Second-order conditions for the existence of local optimality.
- 3.3- Global optimums: Convex programs and Weierstrass Theorem.
- 3.4- Economic interpretation of Lagrange multipliers.

#### Unit 4: Linear programming

- 4.1- Formulation of a linear programming problem.
- 4.2- Solutions of a linear program. Basic feasible solutions.
- 4.3- Characterization of the optimal basic solutions. Simplex algorithm.
- 4.4- Introduction to sensitivity analysis.
- 4.5- Introduction to the dual program.

#### Unit 5: Introduction to ordinary differential equations

- 5.1- Introduction to dynamic analysis.
- 5.2- Concept of differential equation, solution and types of solutions.
- 5.3- First order ordinary differential equations:
  - 5.3.1. Equations in separate variables.
  - 5.3.2. First order linear equations.
- 5.4- Linear differential equations of order  $n$  with constant coefficients.
- 5.5- Qualitative analysis: break-even points and stability.

### 4. Academic activities

The following activities are proposed:

**Master classes:** 30 hours. It will combine the presentation of concepts, results and participatory resolution of exercises, where the theoretical aspects will be applied immediately. Classes will be face-to-face and will be taught to the whole group.

**Practical classes:** 30 hours, solving exercises and problems of an economic nature with the help of the teacher. The exercises will be available at the [url](#). Classes will be face-to-face and will be taught to half of the group.

**Personal work:** 84 hours

- **Teaching assignments:** up to 24 hours, in which various activities directed and reviewed by the faculty may be carried out.
- **Study:** from 60 hours.

**Assessment tests.** 6 hours

6 ECTS = 150 hours

In principle, the teaching methodology and its evaluation is planned to be based on face-to-face classes . However, if circumstances so require, they may be carried out online.

## 5. Assessment system

The assessment will be global, both in first and second call, and will consist of a final exam to be taken in the period established by the Centre. This test will be written and will evaluate the proposed learning results by means of theoretical, practical and/or theoretical-practical questions that will be adjusted to the subject matter. Grading out of 10 points.

In addition, in the **first call**, there is the possibility of **taking a voluntary midterm test** valued at 5 points. This test will assess the knowledge on the subject corresponding to units 1, 2 and 3 of the program, and will be carried out on the date and place that the teacher, with sufficient advance notice, will indicate in the classroom and/or teaching platforms. Students who obtain in this test a grade higher or equal to 50% of the grade (2.5 points out of 5) may choose to eliminate these topics from the global exam of the first call and take only the remaining topics (valued at 5 points). IN this case, the grade corresponding to the eliminated topics will be transferred to the grade of the global exam. To pass the subject the student must obtain a minimum of 5 points out of 10. In order to be eligible for this form of assessment it is mandatory to actively participate and solve the questions, exercises and tests that will be carried out in the face-to-face classes according to the indications that the teacher responsible for each group of the subject will expose the day of the presentation. In this case it is necessary to participate in at least in 75% of the proposed activities.

### Evaluation criteria

It will be assessed whether the student has acquired the learning results outlined above. In particular, the following aspects will be assessed:

1. The correct use of writing mathematical language.
2. Logical reasoning in the approach and resolution of problems.
3. The reference to the theoretical content used is noteworthy.
4. The choice of the appropriate method for solving the problem.
5. Clarity in the application of mathematical concepts and procedures.
6. The correct expression in the results obtained when solving problems.
7. Interpretation of the results in the context of the problem posed, if applicable.