Universidad Zaragoza

Academic Year/course: 2023/24

27035 - Fourier Analysis

Syllabus Information

Academic year: 2023/24
Subject: 27035 - Fourier Analysis
Faculty / School: 100 - Facultad de Ciencias
Degree: 453 - Degree in Mathematics
ECTS: 6.0
Year: 4
Semester: Second semester
Subject type: Optional
Module:

1. General information

This is an elective course within the degree of Mathematics program. Its objective is to introduce the student to the fundamentals of Fourier analysis.

The expression of a periodic function as a Fourier series allows us to study its properties from that of its coefficients. We will analyze the equality function=series with techniques similar to other series expansions learned in the degree. We will study from the functional point of view the Fourier transform and some of its applications (uncertainty principle). Finally, the discretization of the procedures will allow us to use a computer to remove noise or correct drawings, in short, to use filters on signals.

It is necessary to have passed the subjects Mathematical Analysis I, Mathematical Analysis II, Complex Variable and Lebesgue Integral. It is recommended to have passed Functional Analysis. The course requires a good knowledge of the Lebesgue integral and the L^1 and L^2 spaces.

The approaches and objectives of this module are aligned with the Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda; the learning activities could contribute to some extent to the achievement of the goals 4 (quality education), 5 (gender equality), 8 (decent work and economic growth), and 10 (reducing inequality).

2. Learning results

- Know that a periodic function is determined by its Fourier coefficients and understand some convergence results of Fourier series.
- Know how the Fourier coefficients can be obtained by the discrete Fourier transform, and use the basics of the fast Fourier transform.
- Adapt the theory to non-periodic functions with the Fourier transform and understand some inversion results.

3. Syllabus

1. **Historical, physical and mathematical introduction.** The vibrating string and the wave equation: D'Alembert, Euler, and Bernoulli. The heat transmission and its equation: Fourier. The concept of function: measure theory and functional analysis. The electromagnetic waves.
2. **Preliminary mathematics.** Banach spaces of continuous, differentiable, and integrable functions. Convergence of sequences and series of functions. Periodic functions, the torus and a little bit of complex analysis.
3. **Fourier series.** Formal Fourier sine, cosine and exponential series. Statement of the problem of the convergence of Fourier series: convolution, kernels, the unit circle, and its relation with complex analysis and the involved spaces. Pointwise, uniform and mean convergence results: summabilities of Fourier series. Riemann-Lebesgue lemma. Dirichlet's theorem and the Gibbs phenomenon. Riemann's localization principle. Exploiting the orthogonality: Hilbert spaces and Plancherel's theorem.
4. **Discrete Fourier transform.** Periodic sequences. The discrete transform and its inverse. Sampling and interpolation. Approximate calculus of Fourier coefficients. The FFT algorithm and its use in computer programs (Python).
5. **Fourier transform.** The continuous analog of Fourier series. Continuous frequencies. The Schwartz class of functions. Poisson and Gauss-Weierstrass kernels. The inversion formula. Fourier transform and L^2 theory. Band limited functions. The uncertainty principle.

4. Academic activities

Master classes: 30 hours.
Problem solving: 20 hours.
Computer classes: 10 hours.
Study: 83 hours.
Assessment tests: 7 hours.

5. Assessment system

As a general rule, the module can be passed either showing a regular work along the academic year, or by a final exam.

- Regular work. During the course, the student results will be evaluated through a periodical supply of exercises or short tasks, together with their active participation during the course. The use of LaTeX in written presentations is recommended; the evaluation can also include oral presentations. These evaluations will constitute 100% of the final mark.
- Final exam. The aforementioned procedure does not exclude the right, according to the current regulations, to a final exam which, by itself, allows to pass the module.