

27034 - Functional Analysis

Syllabus Information

Academic year: 2023/24

Subject: 27034 - Functional Analysis

Faculty / School: 100 - Facultad de Ciencias

Degree: 453 - Degree in Mathematics

ECTS: 6.0

Year: 4

Semester: First semester

Subject type: Optional

Module:

1. General information

This purpose of this course is the study of the properties of infinite-dimensional vector spaces given by vector spaces of functions endowed with a norm. Special emphasis will be put in the study of Hilbert spaces, in which the norm comes from a scalar product, obtaining a concept of orthogonality and extending many properties of the Euclidean norm to the infinite-dimensional case.

The approaches and objectives of this module are aligned with the Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda; the learning activities could contribute to some extent to the achievement of the goals 4 (quality education), 5 (gender equality), 8 (decent work and economic growth), and 10 (reducing inequality).

2. Learning results

Students will reach a good comprehension of mathematical analysis in its deep connection with algebra and topology, culminating in this way the vision of analysis in the degree in mathematics. In particular, they will get to:

- Know the analytic and geometric forms of the Hahn-Banach theorem and its main consequences.
- Comprehend what completeness implies in relation to normed spaces, continuous and linear maps in this kind of spaces, and the spaces with a scalar product.

3. Syllabus

1. Normed and Banach spaces.
2. $L_p(\mu)$ spaces.
3. Modes of convergence of functions sequences.
4. Hilbert spaces.
5. Spectral theory of compact self-adjoint operators in Hilbert spaces on \mathbb{C} .
6. The fundamental theorems of functional analysis: the Hahn-Banach theorem, the open mapping theorem, and the Banach-Steinhaus theorem.

4. Academic activities

Master classes: 40 hours.

Problem solving: 20 hours.

Study: 83 hours.

Assessment tests: 7 hours.

5. Assessment system

The assessment will be made by a continuous evaluation system, which will consist of 4 tests, which will be rated, each one of them, over 25 points. The final mark, over 100 points, will be the sum of the marks obtained in these tests. The course will be passed with a final mark of 50 or higher.

The date of each one of these 4 tests will be fixed early enough and, in case that they need to be done out of the regular lecture hours, it will be guaranteed that all the students can take them.

Each one of these tests will consist both in theoretical questions, which will consist on questions about definitions or proofs of

results seen in class, as well as practical exercises, which will consist on the resolution of exercises similar to those treated in the lecture room and in the material provided by the instructor.

The criteria in the assessment will take into account the ability to provide precise definitions and correct proofs of the main results treated in the course, as well as the ability to solve different problems, in a correct way and relying on the results and definitions seen in the course.

The students will have the right of taking a global exam, in the dates of the official convocations, fixed by the Faculty of Science.