Universidad Zaragoza

Academic Year/course: 2023/24

27016 - Probability

Syllabus Information

Academic year: 2023/24
Subject: 27016 - Probability
Faculty / School: 100 - Facultad de Ciencias
Degree: 453 - Degree in Mathematics
ECTS: 6.0
Year: 3
Semester: First semester
Subject type: Compulsory
Module:

1. General information

It is a course dedicated to the study of random variables in uncertain environments and the construction of stochastic models that represent real-life situations.

Basic knowledge of linear algebra and mathematical analysis (calculus in one and multiple variables) is required. As it is a continuation of the *Introduction to Probability and Statistics* course, it is crucial to have taken it beforehand.

The approaches and objectives of this module are aligned with the Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda; the learning activities could contribute to some extent to the achievement of the goals 4 (quality education), 5 (gender equality), 8 (decent work and economic growth), and 10 (reducing inequality).

2. Learning results

- Distinguish deterministic phenomena from random phenomena. Understand the statistical paradigm used to study random phenomena: the probability space.
- Construct the appropriate probability space for the study of a random phenomenon. Calculate probabilities. Understand and interpret the concepts of stochastic dependence and independence. Calculate probabilities in both dependent and independent situations.
- Define and understand the meaning of discrete and continuous random variables, both unidimensional and multidimensional, and the functions that characterize them. Understand their utility in calculating probabilities. Know and calculate the main characteristics of a distribution.
- Understand and apply the basic concepts of convergence of sequences of random variables and some laws of large numbers.
- Know the central limit theorem, understand its meaning, and use it appropriately.
- Use acquired knowledge to construct models that solve situations where randomness is essential.

3. Syllabus

Block 1. Random vectors.

1. General random vectors. Definition. Cumulative distribution function. Transforms of random vectors. Types of random vectors.
2. Discrete random vectors. Probability distributions: joint, marginal, condicional. Independent random variables.
3. Continuous random vectors. Probability distributions: joint, marginal, conditional. Independent random variables. Differentiable transform of a continuous random vector.
4. Moments and properties of random vectors. Moments. Moment generating functions. Reproductive property.
5. Some multivariate probability distributions. Multinomial distribution and Multivariate Normal distribution.
6. Correlation and least mean square principle. Correlation coefficient. Schwarz's inequality. Functional relationships between two random variables and the least mean square principle.

Block 2. Stochastic convergence, law of large numbers and central limit theorem.

1. Convergence of sequences of random variables. Convergence in probability. Almost sure convergence. Convergence

in distribution. Convergence in the Lp-norm. Properties and relationships between the types of convergence.

2. Laws of large numbers. Weak laws of large numbers. Strong laws of large numbers. Central limit theorem for independent and identically distributed random variables. General central limit theorem.

4. Academic activities

Master classes: 30 hours.
Problem solving: 30 hours.
Project: 12 hours.
Study: 72 hours.
Assessment tests: 6 hours.

5. Assessment system

The students may take two mid-term exams corresponding to block 1 and block 2. The first exam, corresponding to block 1 will be held at the end of that block. The exam corresponding to block 2 will be held at the official date of the February exam. A mean score of 5 and a minimum score of 4.5 in the two midterm exams is needed to pass the subject.

In the group taught in English, there will be three 2 hour practical classes where students will present solutions of problems, summaries of theoretical results... These activities are optional and will be assessed and marked between 0 and 1 points; this score will be added to the final score obtained in the exams.

The students may also take a unique final exam in each of the two calls (January and July) of the subject. In the second call, there will be one exam including all the topics in the syllabus.