Universidad Zaragoza

Academic Year/course: 2023/24

27009 - Ordinary Differential Equations

Syllabus Information

Academic year: 2023/24
Subject: 27009 - Ordinary Differential Equations
Faculty / School: 100 - Facultad de Ciencias
Degree: 453 - Degree in Mathematics
ECTS: 9.0
Year: 2
Semester: Annual
Subject type: Compulsory
Module:

1. General information

This is a compulsory course whose objective is to introduce the concept of ordinary differential equations, providing the student with the main tools for the analysis and resolution of this type of equations. The student will be put in contact with real problems that can be described by means of this type of equations.

The approaches and objectives of this module are aligned with the Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda; the learning activities could contribute to some extent to the achievement of the goals 4 (quality education), 5 (gender equality), 8 (decent work and economic growth), and 10 (reducing inequality).

2. Learning results

- Distinguish a differential equation from other types of equations and classify it according to its linearity and other characteristics.
- Analyze the existence, uniqueness and regularity of the solutions.
- Apply the different methods of solving such equations when this is possible and analyze qualitatively the form of the solutions when it is not possible to find the solution.

3. Syllabus

I. Linear systems: constant coefficients.
1. Linear differential equations with constant coefficients.
   - First-order homogeneous equation.
   - First-order nonhomogeneous equation.
2. Linear systems: introduction.
   - Terminology and first properties.
   - Eigenvectors and eigensolutions.
   - Generalized eigenvectors and generalized eigensolutions.
3. Exponential matrix.
   - Convergence.
   - Exponential matrix definition and first properties.
   - Exponential matrix via generalized eigensolutions.
   - Differential of the exponential matrix.
4. Linear systems.
   - Solution of homogeneous system.
   - Solution of a nonhomogeneous system.
   - Higher-order differential equations.
5. Higher-order differential equations.
   - Solution of the homogeneous equation.
   - Solution of the non homogeneous equation.
   - Undetermined coefficients.

6. Qualitative theory.
    - Notion of stability.
    - Stability and spectrum.
    - Phase portrait. Classification of 2-d systems.
7. Laplace transform.
    - Laplace transform defined.
    - Calculus of Laplace transform.
    - Calculus of inverse Laplace transform.
    - Solution of initial value problems.
    - Stability.

II. Linear systems: general case.

1. Linear equations.
    - Homogeneous equations.
    - Nonhomogeneous equations.
    - Grönwall inequality.
2. Linear systems.
    - Existence and uniqueness of solutions (homogeneous system).
    - Superposition principle. Resolvent matrix.
    - Nonhomogeneous equations.
    - Higher-order equations.
    - Stability.*
3. Periodic systems.*
    - Periodic solutions.
    - Structure of the solution.
    - Stability and resonance.

III. Nonlinear systems.

1. Autonomous equations.
    - Some examples and properties.
    - Existence and uniqueness. Asymptotes.
    - Qualitative analysis.
2. Nonautonomous equations.
    - Exact equations.
    - Integrating factors.
    - Other methods (separable, homogeneous...).
3. Existence and uniqueness.
    - Lipschitz functions.
    - Existence and uniqueness: Picard theorem.
    - Maximal solution.
    - Global solution.
4. Numerical methods.
    - Euler methods. Taylor method.
    - Convergence.
    - Runge-Kutta method.
    - Multistep methods.*
5. Regularity of the general solution.
    - Continuous dependence.
    - Smooth dependence.
    - The variational equation.
    - Trivialization.*
6. Qualitative theory.
    - Autonomous systems.
    - Stability of equilibria: linearization method.
    - Stability of equilibria: Lyapunov functions.*
    - Phase diagram.

Topics marked with * will be included only if time allows.

4. Academic activities

Master classes: 60 hours.
Problem solving: 30 hours.
Project: 40 hours.
Study: 90 hours.
Assessment tests: 5 hours.

5. Assessment system

There will be a theory and problems exam at the end of each semester, on the dates established by the Faculty of Sciences. This section will represent 80% of the grade. To pass the course it will be necessary to pass each of these exams separately. These grades will be kept for the second call, if applicable.

The student's learning will be evaluated by solving problems and theoretical-practical questions proposed by the teacher in small groups throughout the course. The results will be presented by the students and discussed in class. This section will represent 15% of the grade. This grade will be kept for the second call, if applicable.

A report with the results of a group work must be handed in within the term established by the professor. This section will represent 5% of the grade. This grade will be kept for the second call, if applicable.

According to the current regulations, the student who wishes to do so, may choose to opt for a global test of the subject in the official exams.