

27005 - Graphs and Combinatorics

Syllabus Information

Academic year: 2023/24

Subject: 27005 - Graphs and Combinatorics

Faculty / School: 100 - Facultad de Ciencias

Degree: 453 - Degree in Mathematics

ECTS: 6.0

Year: 1

Semester: Second semester

Subject type: Compulsory

Module:

1. General information

When solving different scientific problems, some combinatorial and graph theory questions arise, and it is important to know how to recognize and solve them. The main objective of this course is then to present basic techniques in the combinatorial analysis, as well as methods and algorithms to solve basic problems in graph theory.

The approaches and objectives of this module are aligned with the Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda; the learning activities could contribute to some extent to the achievement of the goals 4 (quality education), 5 (gender equality), 8 (decent work and economic growth), and 10 (reducing inequality).

2. Learning results

- Solve elementary problems of ordering and enumeration.
- Know and manage the concept of generating function and the recurrence formula.
- Use generating functions to obtain formulas for enumeration problems.
- Know the basics of graph theory and its most elementary applications.
- Apply the basic algorithms of graph theory to solve problems.

3. Syllabus

Section I

1. Enumerative combinatorics: permutations and combinations.
2. Binomial coefficients.
3. Recurrence relations. Some applications.
4. The inclusion-exclusion principle. Applications.

Section II

5. Generating functions.
6. Rational generating functions.

Section III

7. Graphs: definitions and notation.
8. Traversing a graph. Algorithms BFS and DFS.
9. Applications of graph traversal: connected components, strong components, bases.
10. The number of trees and paths of a graph.

Section IV

11. Weighted graphs. Algorithms for the minimum spanning tree problem.
12. The shortest path problem. Dijkstra's algorithm.
13. PERT-CPM algorithms for scheduling a set of project activities.

Section V

14. Maximum flow in a network.
15. The Ford-Fulkerson method for calculating a maximum flow.
16. Menger's theorems on connectivity of graphs.
17. Maximum matching in bipartite graphs. Hall's theorem.
18. Some NP-Hard problems on graphs.

4. Academic activities

Master classes: 30 hours.
Problem solving: 30 hours.
Study: 84 hours.
Assessment tests: 6 hours.

5. Assessment system

There will be one 2-hour midterm exam, only on combinatorics topics. Each student will get a mark, P_1 , in the range 0 to 10.

In the 4-hour final exam, there will be two parts: One with questions on combinatorics and the other one with questions about graph theory. The marks obtained in each part, E_1 and E_2 , will be also in the range 0 to 10.

The final grade, C , will be either $C = 0.5 \cdot (E_1 + E_2)$ if $P_1 < 4$, or $C = 0.5 \cdot (\text{Max}(P_1, E_1) + E_2)$ if $P_1 \geq 4$.

The grade required to pass the course is 5 or greater.