Universidad Zaragoza

Academic Year/course: 2022/23

27608 - Mathematics II

Syllabus Information

Academic Year: 2022/23
Subject: 27608 - Mathematics II
Faculty / School: 109 - Facultad de Economía y Empresa
Degree: 450 - Degree in Marketing and Market Research
ECTS: 6.0
Year: 1
Semester: Second semester
Subject Type: Basic Education
Module:

1. General information

1.1. Aims of the course

The general objectives of the mathematical subjects in this Degree are included in the following two main goals: (1) Mathematical education, (2) Training to apply Mathematics to the challenges that the students will encounter in their careers.

The subject Mathematics II supposes a step forward in these objectives which Mathematics I also dealt with. Education in Mathematics is significant not only because of the transmission of new concepts, but also because the students gain a rigorous and accurate perspective, as well as the capacity for abstraction and the scientific method that characterize Mathematics. Regarding the second goal, this subject introduces students to modelling, using the mathematical analysis approach through two different ways: classical optimization, and dynamical analysis.

These objectives are aligned with the Sustainable Development Objectives (ODS) of the UN Agenda 2030 and specific goals (https://www.un.org/sustainabledevelopment/es/), so that the acquisition of the learning outcomes of the subject provides training and competence to contribute to some extent to their achievement.

Objectives 1-17 (in all of them mathematical modelizations can be formulated).

1.2. Context and importance of this course in the degree

As Mathematics is a tool and a support for other subjects that are essential in the education of the students (Microeconomics, Macroeconomics, Econometrics, etc.), Mathematics II continues the line of work of Mathematics I by bringing Mathematics closer to problems in economic scenarios, which will undoubtedly facilitate a deeper comprehension of and, as a consequence, better skills in applying Mathematics.

After passing the mathematical subjects in the Degree, the students will have worked towards attaining one of the most important goals of mathematical theory: to formulate models that explain the real world. Prospective graduates will be able to use the language of science and to understand the role played by Mathematics in the development of their thinking skills, given that the students? logical reasoning, accuracy, rigor, capacity for abstraction and skills in interpreting results will be improved. This is why the subjects of Mathematics are indispensable tools which allow the designing of appropriate models that are used for researching, describing, understanding and thinking about the realities of companies

1.3. Recommendations to take this course

The students should have a good command of all the contents of the subject Mathematics I, taught during the first semester of the first year. They must, in any case, know the meaning and implications of the differentiability of a function and be skilled in the calculus of partial derivatives. The students also have to know how to determine the sign of a quadratic form. They must also be able to present and support an argument with a logical sequence and to connect various mathematical aspects previously learnt.

2. Learning goals

2.2. Learning goals

1. **The student, in order to pass the course, will have to show her/his**

competence in the following skills:

1. To have gained good skills in using mathematical language, both in comprehension and writing.
2. To be able to identify the fundamental elements of an optimization problem: variables, objective function and constraints.
3. To be able to formulate static optimization problems: unconstrained, and with equality and/or inequality constraints.
4. To know how to solve an optimization problem by the graphical method, when that is possible.
5. To be able to evaluate whether or not a mathematical programmer meets the conditions that allow it to be solved by the techniques learnt.
6. To be able to distinguish between critical points and extrema (optima).
7. To be able to discriminate between local and global optima.
8. To be able to distinguish between necessary conditions and sufficient conditions for local optimality.
9. To be able to calculate the critical points by solving the system of equations obtained by applying the first-order conditions for local optimality, both for unconstrained cases and for problems with equality constraints.
10. To know how to classify the obtained critical points by using the second-order conditions, both for unconstrained optimization programs and for problems with equality constraints.
11. To be able to apply the conditions which guarantee that an optimum is global.
12. To be able to interpret economically the Lagrange multipliers obtained in an optimization problem with equality constraints.
13. To be able to evaluate whether a mathematical program is linear. If it is, they must know how to solve it by the graphical method (when that is possible) and by the simplex algorithm.
14. When varying a parameter of a linear optimization program, the students must be able to analyze how the solution changes, without solving the new problem.
15. To be able to use some computer programs to find the solution to an optimization problem and to be able to interpret the results obtained.
16. To be able to identify a dynamic process in an economic scenario and be able to represent this process (when possible) by an ordinary differential equation.
17. To understand the concept of the solution of an ordinary differential equation and to be able to distinguish between general solution and particular solution.
18. To be able to discriminate between a first-order differential equation and a linear differential equation of order n.
19. To be able to identify whether a first-order differential equation is with separable variables, homogeneous, exact, or of linear type, and to know how to solve the equation by the appropriate method.
20. For a linear differential equation with constant coefficients, they must be able to write the complementary (homogeneous) equation and obtain its general solution.
21. To be able to find a particular solution of a linear differential equation with constant coefficients.
22. To have the know-how to calculate the general solution of a linear differential equation with constant coefficients.
23. To be able to work out the solution of a linear differential equation of order n with constant coefficients, given n initial conditions.

2.3. Importance of learning goals

They permit the comprehension of theoretical concepts and models that are part of the contents of other related subjects studied in the Degree. Mathematics is most important in this goal because it facilitates the analysis and discussion of the models and concepts studied. In this regard, it is worth mentioning that Optimization techniques allow the laying of the foundations of the two basic paradigms of Microeconomics, namely, the theory of consumer choice and the production theory. The concepts of convex set and concave/convex function, whose economic interpretations are, respectively, the diversity in consumption and the law of diminishing marginal returns, have important applications. Linear Programming is very useful in production planning problems and it allows the solving of some simple exercises of comparative statics.

Different techniques are required for the analysis of dynamic processes in continuous time, which is essential, for example, in models of economic growth. The theory of differential equations provides the necessary tools to deal with some key concepts such as trajectory over time, evolution of the system, stability, etc.

3. Assessment (1st and 2nd call)

3.1. Assessment tasks (description of tasks, marking system and assessment criteria)

The student will prove that he/she has achieved the expected learning results by means of the following assessment tasks:

The evaluation will be global in both the first and second sittings. It will consist of a final exam to be taken on the dates determined by the Faculty. The global exam will be written and will assess the proposed learning outcomes through questions that are theoretical, practical, or of a mixed theoretic-practical character and that will be based on the topics taught. It will be worth 10 points.

In addition, in the first sitting, it will be possible to take a voluntary intermediate test. This test will assess the student?s knowledge about the topics of the subject belong Part I of syllabus

The above mentioned test will take place in the half of period, and the date will be announced in advance in class and/or in the virtual teaching platform.

The students who obtain a mark of at least 40% of the maximum in this test will be able to eliminate the corresponding topics from the global exam at the first sitting. In this case, the mark corresponding to the eliminated topics will be added to the mark of the global exam in a weighted fashion. This weight will be about 50% of the global value (10 points). In order to pass the subject, students have to obtain at least 5 points out of 10.

It has to be taken into account that the evaluation process closes at the end of the academic year, so it is not possible to claim academic merits from one academic year in a later one.

Students taking their exams at the fifth or sixth opportunity will be marked following the rules established under the Governing Council Agreement on 22 December 2010, which sets out the assessment regulations in the University of Zaragoza.

Evaluation Criteria

Students will be assessed on whether they have acquired the learning outcomes mentioned above. In particular, they will be assessed on the following aspects:

1. Correct mathematical writing.
2. Logical reasoning in the posing and solving of the problems.
3. Reference to the theoretical results used, when relevant.
4. The choice of the most appropriate method for the solving of problems.
5. Clarity in the application of mathematical concepts and procedures.
6. Computations carried out with care.
7. The correct expression of the results obtained when solving problems.

Students taking their exams at their fifth or sixth opportunity will be marked following the rules established under the Governing Council Agreement on 22 December 2010, which sets out the assessment regulations in the University of Zaragoza.

4. Methodology, learning tasks, syllabus and resources

4.1. Methodological overview

The objective of this subject is that the students should develop the analytical skills, rigour and intuition needed for using mathematical concepts and results and that they should be able to apply these abilities to the analysis of problems of an economic nature. Therefore, the teaching should aim to provide students with a solid mathematical knowledge and to train them in a way of reasoning that will allow them thereafter to successfully solve a wide variety of questions in an economic scenario.

The teaching methodology is planned for face-to-face classes. However, if necessary for health reasons, teaching could be delivered on line

4.2. Learning tasks

The program offered to the students to help them achieve the learning results includes the following activities**:**

- **Type 1 activities Lectures** (1.2 ECTS: 30 hours) which will be based on lectures to present the concepts and results corresponding to the contents. At the same time, some exercises will be solved with the participation of the students to help them comprehend the theoretical concepts presented.
- **Type 2 activities Practical lessons in classroom** , (1.2 ECTS: 30 hours) in which the students will apply the theoretical results in order to solve, with the teacher?s help, more complete exercises, and problems of an

economic nature. Problem sheets will be available for the students and the teacher will announce in advance the problems that will be solved in each practical lesson so that the students can prepare them beforehand.

- **Type 6 activities Teaching assignments** (Up to 24 hours, 0.96 credits), which may consist of a number of different activities designed to support the learning process, including: follow-up of some simple projects that had been assigned to small teams of students and the presentation of these projects; collective tutorials on specific subjects, the use of which will be recorded in a document ; solving problems of an economic nature by using some of the mathematical tools taught during the classes, etc. These activities may also be devoted to the teaching of more advanced topics, intended for the students interested in learning some further mathematical tools that would allow them to deal with more general problems. In this way, the students are shown that both Mathematics and Economics are vibrant sciences with many facets to be studied.
    - **Type 7 activities Autonomous work and study** (from 60 hours).
    - **Tyoe 8 activities Assessment**. Final exam and midterm exams: (6 hours)

Total: 150 hours (6 credits ECTS)

If the availability of teaching staff is less than the teaching assignment and type 6 activities cannot be carried out, type 7 activities shall be substituted.

The teaching methodology is expected to be face-to-face. However, if for public health reasons it were necessary, the classes might take place online.

4. Out of class work: 3.6 ECTS credits

The assessment will be prepared to be carried out as face-to-face examination, but if health circumstances do not allow it, they will be carried out by doing it entirely online or in a blended way. In the case of online exams, it is important to highlight that, the student may be recorded, and he or she can exercise his or her rights by the procedure indicated in.

https://protecciondatos.unizar.es/sites/protecciondatos.unizar.es/files/users/lopd/gdocencia_reducida.p

The necessary software will be used to check the possibility of plagiarism. The detection of plagiarism or copying in an activity will imply that the activity or exam will be marked 0/10.

4.3. Syllabus

TABLE OF CONTENTS

Chapter 1: Mathematical programs

1.1. General formulation of a mathematical program. Classification.

1.2. Definitions and properties. Weierstrass? Theorem.

1.3. Graphical solving.

1.4. Introduction to convexity:

1.4.1. Convex sets. Definition and properties.

1.4.2. Convex and concave functions. Definitions and properties.

1.4.3. Convex programs.

Chapter 2: Programming without constraints

2.1. Problem?s formulation.

2.2. Local optima:

2.2.1. First order conditions for the existence of a local optimum.

2.2.2. Second order conditions for the existence of a local optimum.

2.3. Global optima: convex programs.

Chapter 3: Programming with equality constraints

3.1. Problem?s formulation.

3.2. Local optima:

3.2.1. First order conditions for the existence of a local optimum.

3.2.2. Second order conditions for the existence of a local optimum.

3.3. Global optima: convex programs and Weierstrass? Theorem.

3.4. Economic interpretation of the Lagrange?s multipliers.

Chapter 4: Linear programming

4.1. Formulation of a problem of linear programming.

4.2. Solutions of a linear program. Basic feasible solutions.

4.3. Characterization of the optimal basic feasible solutions. Simplex? Algorithm.

4.4. Introduction to the sensitivity analysis.

4.5. Introduction to the dual program.

Chapter 5: Introduction to ordinary differential equations

5.1. Introduction to the dynamical analysis.

5.2. Concept of differential equation, solution and types of solution.

5.3. First order ordinary differential equations:

5.3.1. Separable equations.

5.3.2. Linear first order equations.

5.4. Linear differential equations of order n with constant coefficients.

5.5. Qualitative analysis: equilibrium points and stability.

4.4. Course planning and calendar

The calendar will be explained to the students in the presentation of the subject.

4.5. Bibliography and recommended resources

[BB: Basic bibliography / BC: Complementary bibliography]

- [BB] Balbás de la Corte, Alejandro. Análisis matemático para la economía. II, Cálculo integral y sistemas dinámicos / Alejandro Balbás de la Corte, José Antonio Gil Fana, Sinesio Gutierrez Valdeón Madrid : AC, 2005
- [BB] Balbás de la Corte, Alejandro. Programación matemática / Alejandro Balbás, Jose Antonio Gil . - 2a. ed, 3a. reimp. Madrid : AC, 2005
- [BB] Barbolla García, Rosa.. Optimización : programación matemática y aplicaciones a la economía / Rosa Barbolla, Emilio Cerdá, Paloma Sanz. . - 1ª ed., 1ª imp. Madrid : Garceta, 2011.
- [BB] Barbolla, Rosa. Optimización : cuestiones, ejercicios y aplicaciones a la economía / Rosa Barbolla, Emilio Cerdá, Paloma Sanz . - [1a. ed. en español], reimp. Madrid [etc.] : Prentice Hall, 2006
- [BB] Blanco García, Susana. Matemáticas empresariales I : enfoque teórico-práctico. Vol. 2, Cálculo diferencial / Susana Blanco García, Pilar García Pineda, Eva del Pozo García . - [1ª ed.] Madrid : Editorial AC, [2004]
- [BB] Calderón Montero, Susana. Matemáticas para la economía y la empresa / Susana Calderón Montero, María Lourdes Rey Borrego ; Colaboradores Teodoro P. Galache Laza, Francisco Ruiz de la Rúa Madrid : Pirámide, 2012
- [BB] Pérez Grasa, Isabel. Matemáticas para la economía : programación matemática y sistemas dinámicos / Isabel Pérez-Grasa, Esperanza Minguillón Constante, Gloria Jarne Jarne Madrid [etc] : McGraw-Hill, cop. 2001
- [BB] Programación matemática y modelos económicos : un enfoque teórico-práctico / Antonio Heras Martínez... [et.al.] Madrid : AC, D. L. 1990
- [BC] Mocholi Arce, Manuel. Programación lineal : ejercicios y aplicaciones / M. Mocholi Arce, R. Sala Garrido Madrid : Tebar Flores, D.L. 1984
- [BC] Zabal Cortés, Trinidad. Matemáticas II : Manual para los grados de carácter económico. Curso 2015-2016 / Trinidad Zabal. Zaragoza : Universidad, Facultad de Economía y Empresa, Taller de Edición e Impresión, Cop. 2016

Webs:

- Actividad de aprendizaje a través del descubrimiento de errores en la asignatura "Matemáticas II" de los grados de carácter económico [http://www.unizar.es/aragon_tres/MatII/erroresMII.htm]