

27040 - Topology of Surfaces

Syllabus Information

Academic Year: 2022/23

Subject: 27040 - Topology of Surfaces

Faculty / School: 100 - Facultad de Ciencias

Degree: 453 - Degree in Mathematics

ECTS: 6.0

Year: 4

Semester: Second semester

Subject Type: Optional

Module:

1. General information

1.1. Aims of the course

This subject and its syllabus have the following goals:

Give the student a topological sense of the study and classification of surfaces. The notion of topological invariant, such as the fundamental group, is relevant to the study of mathematical objects. In this class, a particular topological invariant, having an algebraic structure (a group) will be able to determine the topological structure of compact surfaces, and even determine their orientability.

These approaches and objectives are aligned with the following Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda (<https://www.un.org/sustainabledevelopment/es/>), in such a way that the acquisition of the learning outcomes of the module provides training and competence to contribute to some extent to their achievement: (4) Quality education, (5) Gender equality, (8) Decent work and economic growth, (9) Industry, innovation and infrastructure, (10) Reducing inequality, (17) Partnerships for the goals.

1.2. Context and importance of this course in the degree

This subject is part of the módulo *Ampliación de Geometría y Topología* (Higher Geometry and Topology)

As mentioned in section 1.1, it is recommended that the student is familiar with both algebraic and topological techniques, such as those provided in *Algebra lineal*, *Topología general*, and *Estructuras algebraicas*. This class will connect them considering certain topological invariants of an algebraic nature and applying them to solve concrete problems.

1.3. Recommendations to take this course

Students are recommended to have acquired the competences associated with the *Fundamentos de Geometría y Topología* (Fundamentals in Geometry and Topology), in particular *Algebra lineal*, *Topología general* and *Estructuras Algebraicas*.

2. Learning goals

2.1. Competences

Upon successful completion of this subject the student will improve the following abilities:

- Carry out the goals described in section 2.1.
- CG3. To have the ability to gather and interpret the relevant data, particularly in the field of mathematics, in order to make statements using analytical methods as well as abstraction, containing insights on relevant topics, be it of a social, scientific, or ethical nature.
- CG5: To develop learning skills that will be necessary to continue studies in mathematics with a high degree of autonomy.
- CT1. Be able to clearly state, both orally and in writing, the student's reasoning, problem solving techniques, reports, etc.
- CE1. Understand and apply both mathematical language and methods. Learn rigorous proofs of the basic theorems

in the different areas of mathematics.

2.2. Learning goals

In order to pass this module, the student should be able to show the following skills:

- Understand the notion of fundamental group and be able to compute it in some concrete situations.
- Topologically recognize compact surfaces and classify them.

2.3. Importance of learning goals

The learning objectives provide basic skills within the degree (see Context and importance of this course in the degree).

3. Assessment (1st and 2nd call)

3.1. Assessment tasks (description of tasks, marking system and assessment criteria)

The student must demonstrate that they have achieved the learning objectives by means of the following evaluation activities:

- Along the course, students are asked to solve different activities (mostly exercises and problems). These activities are the part of continuous evaluation.
- Besides, the students are asked to prepare a topic for the course, and if the schedule allows it, give an oral presentation about it.
- The final grade will be obtained averaging the degrees of all those tasks.
- The students can take a written exam after the end of the classes. In that case, the final grade will be highest of the two grades.

4. Methodology, learning tasks, syllabus and resources

4.1. Methodological overview

The methodology followed in this course is oriented towards the achievement of the learning objectives. A wide range of teaching and learning tasks are implemented, such as lectures, problem-solving sessions, tutorials and autonomous work and study.

4.2. Learning tasks

This course is organized as follows:

- **Lectures.** Three weekly sessions.
- **Problem-solving sessions** in small groups. One weekly session. Oral presentations of problems.
- **Tutorials.**
- **Autonomous work and study.** In addition to the general teaching methodology activities students are afforded the opportunity to submit individual homework assignments. These assignments are checked by the teacher and returned on a regular basis. This process allows students to pinpoint strengths/weaknesses and helps in their learning process.
- **Final presentations.** Besides presenting their work, each student should do a self evaluation, and an evaluation of the other classmates presentations.

These tasks will take place in-person at the classroom, unless the University of Zaragoza establishes that, because of the public health situation, they should be done online.

4.3. Syllabus

- **Topic 1. Fundamental group.**
 - Definition and preliminaries.
 - Calculations of fundamental groups.
 - The fundamental group of the circumference.
 - Seifert-Van Kampen theorem.
- **Topic 2. Classification of surfaces.**

- Connected sum. Surgery.
- Triangulation. Euler characteristic.
- Classification theorem.
- **Topic 3. Covering spaces.**
 - G-spaces and group actions.
 - Definition and motivation of covering space.
 - Covering spaces of surfaces.
- **Topic 4. Introduction to knot theory.**

4.4. Course planning and calendar

The calendar of classes and oral presentations will be announced previously in class and in the ADD.

The deadlines for the exercises will be announced in class enough time ahead.

Oral presentations will be done in the last two weeks of the class period. The exact times of these presentations will be decided depending on the number of students.

The final exam, if necessary, will take place according to the School academic calendar.

4.5. Bibliography and recommended resources

Basic bibliography:

- Massey, William S.. Introducción a la topología algebraica / William S. Massey . Barcelona[etc.] : Reverté, cop.1982.
- Armstrong, M.A.. Topología básica / M.A. Armstrong . Barcelona [etc.] : Reverté, D.L. 1987.

<http://psfunizar10.unizar.es/br13/egAsignaturas.php?codigo=27040>