Universidad Zaragoza

Academic Year/course: 2022/23

27017 - Galois Theory

Syllabus Information

Academic Year: 2022/23
Subject: 27017 - Galois Theory
Faculty / School: 100 - Facultad de Ciencias
Degree: 453 - Degree in Mathematics
ECTS: 6.0
Year: 3
Semester: First semester
Subject Type: Compulsory
Module:

1. General information

1.1. Aims of the course

This is a compulsory subject in the degree of Mathematics of the University of Zaragoza.

Its main aim is to introduce the students to the basic aspects of group theory and of Galois theory, which uses group theory to study field extensions and algebraic equations.

These approaches and objectives are aligned with the following Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda (https://www.un.org/sustainabledevelopment/es/), in such a way that the acquisition of the learning outcomes of the module provides training and competence to contribute to some extent to their achievement: (4) Quality education, (5) Gender equality, (8) Decent work and economic growth, (9) Industry, innovation and infrastructure, (10) Reducing inequality, (17) Partnerships for the goals.

1.2. Context and importance of this course in the degree

This course presents a key tool in any mathematical area: the theory of groups, which is the tool to measure and take advantage of the symmetries that may appear in any system. It is therefore a basic course.

Inside the syllabus, it corresponds to the algebraic structures module. The student is advised to have completed the second year subject *Algebraic Structures* before enroling in this one.

1.3. Recommendations to take this course

Regular attendance is strongly recommended. Also, the students is expected to particpate actively in the classes, to make use of the office hours to get a better understanding of the subject and also to work regularly on the exercises and problem sheets. The content learnt in the subject *Algebraic Structures* will be very much used so the student is advised to have completed that subject before enroling in this one.

Work in groups and active discussions in class will be incentivated.

2. Learning goals

2.1. Competences

Being succesful in this course should mean that the student is competent to

- Reason in an abstract way.
- Be able to write and communicate abstract concepts of mathematics.
- Be able of autonomous learning.

2.2. Learning goals

To complete this subject, the student must be able of:

- Make computations in some particular groups (cyclic, dihedral, symmetric of small degree) and rings (of numbers, polynomials ans matrices)

- Be familiar with group actions, Sylow theorems and be able to use them to describe the structure of a given group.
- Work with expressions involving algebraic and transcendental elements.
- Compute explicitly some Galois groups.
- Work with Galois correspondence and in particular know the characterization of solvability by radicals of polynomial equations.

2.3. Importance of learning goals

It is a basic part of the syllabus of the Mathematics degree. Moreover, group theory is an important tool in every branch of mathematics.

3. Assessment (1st and 2nd call)

3.1. Assessment tasks (description of tasks, marking system and assessment criteria)

To complete the subject, students will be required to show competence in the required learning goals by means of the following activities.

There will be a partial exam P, graded with a maximum of 10 points, and a comprehensive final exam (during the official period for these exams). The final exam will split into two parts, A and B, each one graded with a maximum of 10 points. The final mark obtained by a student will be the maximum of 0.5P + 0.5B and 0.5A + 0.5B. Students who are successful in the partial exam are allowed not to take part A of the final exam.

The final mark in July will be the mark obtained in July's comprehensive final exam.

4. Methodology, learning tasks, syllabus and resources

4.1. Methodological overview

There will be theory classes with a lot of examples and exercises proposed to students. Teachers will encourage students to participate and to work together to increase both their mathematical communication skills and their ability for abstract reasoning. There will be some group exercises but grading will be based on personal work only. Students will be able and encouaged to attend office hours.

Written notes, exercise sheets will be available in Moodle.

4.2. Learning tasks

Lessons will follow the written notes available for students in Moodle. Proofs will be expanded with details and examples. There will be plently of problem solving, including a detailed solving of the written individual exercises sets used for the grading and also group problem solving.

Classes and evaluation will be in person unless the current sanitary situation makes it necessary to move, to a greater or lesser extent, to on-line teaching. This will be determined in any case by the health authority and the University of Zaragoza.

4.3. Syllabus

1. Groups: basic notions
2. Groups of permutations
3. Group actions and simplicity of A5
4. Rings, Fields, polynomials and existence of roots
5. Field extensions. Algebraic extensions
6. Normal extensions. The Galois group
7. The Galois theorem. Solvable groups. Solving equations by radicals

4.4. Course planning and calendar

There will be four weekly lecture hours. Theoretical and practical parts will not be separated in advance. Exercises and problems, similar to those in the individual written exercises sets will be proposed and discussed. That discussion will also include office hours.

Further information concerning the timetable, classroom, office hours, assessment dates and other details regarding this course will be provided on the first day of class or via Moodle. Date of the comprehensive exam will be determined and made public by the Faculty of Science. For any other question or requirement, students are recommended to contact teachers by email.

4.5. Bibliography and recommended resources

http://psfunizar10.unizar.es/br13/egAsignaturas.php?codigo=27017