

Academic Year/course: 2022/23

27016 - Probability

Syllabus Information

Academic Year: 2022/23

Subject: 27016 - Probability

Faculty / School: 100 - Facultad de Ciencias

Degree: 453 - Degree in Mathematics

ECTS: 6.0

Year: 3

Semester: First semester

Subject Type: Compulsory

Module:

1. General information

1.1. Aims of the course

The objectives of this subject are:

To provide the theoretical knowledge and techniques required in the subjects in the block *Probability and Statistics* of the following courses.

To provide basic tools in probability, which will allow the student to develop models for situations where randomness plays an essential role.

These approaches and objectives are aligned with the following Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda (<https://www.un.org/sustainabledevelopment/es/>), in such a way that the acquisition of the learning outcomes of the module provides training and competence to contribute to some extent to their achievement: (4) Quality education, (5) Gender equality, (8) Decent work and economic growth, (9) Industry, innovation and infrastructure, (10) Reducing inequality, (17) Partnerships for the goals.

1.2. Context and importance of this course in the degree

This subject is included in the block *Probability and Statistics* and it aims to the study of random variables and to the development of stochastic processes able to model real situations.

This subject is a necessary base in order to acquire the necessary knowledge for the remaining subjects in the block *Probability and Statistics* included in the degree of Mathematics. These subjects are Teoría de la Probabilidad, Estadística Matemática and Técnicas de Regresión.

1.3. Recommendations to take this course

Students are assumed to have previous knowledge of the concepts and first properties of probability spaces and (onedimensional) random variables.

The continuous work of the student from the beginning of the course is essential to follow the classes and finally, pass the exam.

We also recommend to work autonomously the proposed exercises and activities. One of the main difficulties of the subject is to know how to build a model which allows us to solve the problems using the tools seen in theory. Consequently, it will be very helpful for the student to dedicate the necessary time to try to solve the problems by himself.

2. Learning goals

2.1. Competences

In order to pass this course, the student will be more competent:

- To identify real situations where the most common probability distributions appear.
- To understand the concepts of independence and conditional distribution
- To calculate probabilities in uncertainty situations.

- To manage and understand the probabilistic language and methods.
- To identify real situations which can be represented with the help of probabilistic models and to build adequate models to that end.
- To learn autonomously new probabilistic theory and methods
- To understand and know how to apply the large numbers laws and the central limit theorem
- To understand the proof of the basic theorems in probability and statistics

3. Assessment (1st and 2nd call)

3.1. Assessment tasks (description of tasks, marking system and assessment criteria)

The students may take two mid-term exams corresponding to block 1 and block 2. The first exam, corresponding to block 1 will be held at the end of that block. The exam corresponding to block 2 will be held at the official date of the February exam. A mean score of 5 and a minimum score of 4.5 in the two midterm exams is needed to pass the subject.

In the group taught in English, T6 activities will consist on three 2 hour practical classes where students will present solutions of problems, summaries of theoretical results... These activities are optional and will be assessed and marked between 0 and 1 points; this score will be added to the final score obtained in the exams.

The students may also take a unique final exam in each of the two calls (February and September) of the subject. In the second call, there will be one exam including all the topics in the syllabus.

4. Methodology, learning tasks, syllabus and resources

4.1. Methodological overview

The methodology followed in this course is oriented towards the achievement of the learning objectives. A wide range of teaching and learning tasks are implemented, such as lectures, problem-solving sessions, laboratory session, seminars, tutorials and autonomous work and study.

A learning process based on critical reflection of concepts is proposed. In order to achieve these objectives, problem based learning is used. The aim of these learning processes is to encourage management of documentary sources, reflection on previously learned theoretical aspects, and structuring a logical discourse from the problem statement to its resolution.

4.2. Learning tasks

This course is organized as follows:

- **Lectures.** They will be based on a participative learning-teaching approach, promoting the interaction with students.
- **Problem-solving sessions.** Small group sessions: Problems and exercises will be worked in in small group classes. Additional exercises will be provided for the student autonomous work.
- **Laboratory sessions/Seminars.** Depending on the time availability, some seminars or computer lab classes will be held to complement the contents developed in class.
- **Tutorials.** Students will have tutorials with the teacher on a schedule to be agreed jointly.
- **Autonomous work and study.** Autonomous work will allow to consolidate the concepts explained in class as well as learn and properly apply the techniques studied. They should handle literature, in addition to class notes. It should also dedicate a significant part of their time to solving the proposed exercises.
- **Assessment tasks.** In addition to the final exam, a midterm exam including topics of section 1 can be optionally done at the end of the class period of that section.

The teaching activities and assessment tasks will take place in a face-to-face mode, except in the case that, due to the health situation, the dispositions emitted by the competent authorities and by the University of Zaragoza compel to take them to a greater or lesser extent in a telematic form.

4.3. Syllabus

Section 1. Random vectors.

- **Topic 1.1.** General random vectors. Definition. Cumulative distribution function. Transforms of random vectors. Types of random vectors.
- **Topic 1.2.** Discrete random vectors. Probability distributions: joint, marginal, conditional. Independent random variables.
- **Topic 1.3.** Continuous random vectors. Probability distributions: joint, marginal, conditional. Independent random variables. Differentiable transform of a continuous random vector.

- **Topic 1.4.** Moments and properties of random vectors. Moments. Moment generating functions. Reproductive property.
- **Topic 1.5.** Some multivariate probability distributions. Multinomial distribution and Multivariate Normal distribution.
- **Topic 1.6.** Correlation and least mean square principle. Correlation coefficient. Schwarz's inequality. Functional relationships between two random variables and the least mean square principle.

Section 2. Stochastic convergence, law of large numbers and central limit theorem.

- **Topic 2.1.** Convergence of sequences of random variables. Convergence in probability. Almost sure convergence. Convergence in distribution. Convergence in the L_p -norm. Properties and relationships between the types of convergence.
- **Topic 2.2.** Laws of large numbers. Weak laws of large numbers. Strong laws of large numbers. Central limit theorem for independent and identically distributed random variables. General central limit theorem.

4.4. Course planning and calendar

There are four class hours per week. In the group taught in English, there will be three optional 2-hour practical classes (T6). Further information concerning the timetable, classroom, office hours, assessment dates and other details regarding this course will be provided on the first day of class or please refer to the Faculty of Sciences website and Moodle.

4.5. Bibliography and recommended resources

- Grimmett, Geoffrey. One thousand exercises in probability / Geoffrey R. Grimmett and David R. Stirzaker . - 1st ed., reprinted Oxford : Oxford University Press, 2003
- Rohatgi, Vijay K.. An introduction to probability theory and statistics / Vijay K. Rohatgi, A. K. Md. Ehsanes Saleh . 3rd ed. New York [etc.] : John Wiley, 2015.
- Vélez Ibarrola, Ricardo. Cálculo de Probabilidades 2 / Ricardo Vélez Ibarrola . - [1ª ed.] Madrid : Ediciones Académicas, 2004

<http://psfunizar10.unizar.es/br13/egAsignaturas.php?codigo=27016>