Academic Year/course: 2022/23

27013 - Geometry of Curves and Surfaces

Syllabus Information

Academic Year: 2022/23
Subject: 27013 - Geometry of Curves and Surfaces
Faculty / School: 100 - Facultad de Ciencias
Degree: 453 - Degree in Mathematics
ECTS: 10.5
Year: 3
Semester: Annual
Subject Type: Compulsory
Module:

1. General information

1.1. Aims of the course

The class will be presented in a way that enables the student to recognize the presence of mathematics in different areas (nature, science, technology, art...). Collaborate in the development of skills associated with the work of the future mathematician

These approaches and objectives are aligned with the following Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda (https://www.un.org/sustainabledevelopment/es/), in such a way that the acquisition of the learning outcomes of the module provides training and competence to contribute to some extent to their achievement: (4) Quality education, (5) Gender equality, (8) Decent work and economic growth, (9) Industry, innovation and infrastructure, (10) Reducing inequality, (17) Partnerships for the goals.

1.2. Context and importance of this course in the degree

This is an interdisciplinary subject where algebra, analysis, topology and geometry go hand in hand. The tools and techniques used are already studied in previous classes to solve geometric problems. The class will provides spatial vision and the scope of application of the results obtained is likely to be used in any other subject of the degree. In particular, the class constitutes a first contact of the student with the field of differential geometry, whose natural continuation can be found in the Geometry and Topology Extension module.

1.3. Recommendations to take this course

To follow this class, it is recommended to have passed the other class of the module (General Topology) as well as having passed the module of Linear Algebra and Geometry and the subjects of Mathematical Analysis I, Mathematical Analysis II and Ordinary Differential Equations. It is recommended to take it simultaneously with Complex Variable and Equations in Partial Derivatives by the interactions between the subjects.

2. Learning goals

2.1. Competences

Upon completing this class, the student will be more competent in the following areas:
Transversal skills:

- CT1 Know how to express clearly, both in writing and orally, reasoning, problems, reports...etc.
- CT3 Distinguish when faced with a problem what is substantial from what is accessory, formulate conjectures and reason to confirm or refute them, identify errors in incorrect reasoning, etc.

Specific skills:

- CE1 Understand and use the mathematical language and method. Know rigorous proofs of the basic theorems of the different branches of mathematics.
- CE2 Propose, analyze, validate and interpret models of simple real situations, using the mathematical tools that are more suitable for the purposes pursued.

- CE3 Solve mathematical problems using basic calculation skills and other techniques.
- CE4 Use computer applications with different types of scientific software to experiment in mathematics and solve problems.

2.2. Learning goals

In order to complete this class successfully, the student must demonstrate the following results:

- Recognize the nature of the points of a curve in $\mathbf{R}^2$ and $\mathbf{R}^3$.
- Calculation of the dihedral and the Frenet trihedral and the curvature and torsion. Fundamental toerems.
- Use the first fundamental form of a surface to solve problems about lengths, angles, and areas.
- Use the second fundamental form of a surface to recognize the nature of its points. Know its relation with the Gauss map. Know how to calculate, apply and interpret the principal, Gaussian and half.
- Understand intrinsic geometric properties: covariant derivative, Gauss's egregious theorem, geodesics and Gauss-Bonnet theorem.
- Understand the difference between local and global problems.

2.3. Importance of learning goals

The learning goals provide the student with knowledge and procedures that are at the base of other subjects. Some of the results obtained (Gauss's egregious theorem, non-Euclidean geometries...) have a fundamental relevance in the history of mathematics.

3. Assessment (1st and 2nd call)

3.1. Assessment tasks (description of tasks, marking system and assessment criteria)

The student must demonstrate that they have achieved the expected learning outcomes through the following evaluation activities:

The evaluation is distributed as follows:

- The written tests have a weight of 60% and a minimum grade of 4/10 is required to pass.
- Computer labs have a weight of 20%.
- The continuous evaluation will have a weight of 20%.
- The student will be able to take an partial test at the end of the first semester. A minimum of 4/10 is required in order to allow averaging with the second partial test of the class.
- In the official calls, the student can decide whether to take a partial test (for the second part of the class) or a final test (both parts of the class). In any case, a minimum of 4/10 is required for each partial test in order to allow averaging.
- The evaluation of the labs will be carried out as follows:
  - a 25% (5% of the total) for attendance and completion of computer labs in class;
  - another 25% (5% of the total) for the evaluation of the problems to be collected;
  - a 50% (10% of the total) for the evaluation of the computer lab's exam.
  - Students who have passed the labs in previous years may choose between maintaining the previous grades or completing the labs again.
- The continuous evaluation will consist of the presentation of problems in class, the completion of assigments or turning in work requested by groups of students on particular topics.

Students who wish to do so may only take a global test in order to assess the acquisition of the class requirements.

4. Methodology, learning tasks, syllabus and resources

4.1. Methodological overview

The methodology followed in this course is oriented towards the achievement of the learning objectives. A wide range of teaching and learning tasks are implemented, such as lectures, problem-solving sessions, laboratory sessions, tutorials and autonomous work and study with the help of moodle.

The learning process that has been designed for this subject is based on the following:

- Lectures.

- Problem sessions.
- Computer labs (use of open source software).
- Group work, study and individual work.

4.2. Learning tasks

This course is organized as follows:

- **In person work** (105 hours). It includes:
  - **Lectures.**
  - **Problem-solving sessions.** Blackboard problem-solving activities. Teamwork involving written and oral presentations. LaTeX prepared texts and use of English is encouraged.
  - **Laboratory sessions.** Computer problem-solving activities using free software (five two-hour sessions).
  - **Office hours.**
- **Independent work and study** (157,5 hours). It includes the study of the class, the completion of exercises proposed, the time dedicated to solving problems and the time derived from the activities carried out in the computer lab.

The teaching activities and assessment tasks will take place in person, or otherwise in compliance with the guidelines approved by the University of Zaragoza due to the health situation.

4.3. Syllabus

The goal of the course *Geometry of curves and surfaces* is the study of the differential geometry of curves and surfaces in the euclidean plane and space.

This course will address the following topics:

- **Topic 1.** Regular plane curves. Frénet's frame, tangent and normal vector fields along a curve, curvature, arc length. Contact theory. Fundamental Theorem for plane curves. Curves as submanifolds.
- **Topic 2.** Biregular spatial curves, Frénet frame (tangent, normal and binormal fields), arc length, torsion, curvature, evolute. Fundamental Theorem for spatial curves. Local canonical form.
- **Topic 3.** Regular surfaces. Local theory: 2-function graphs, charts and regular values of 3-functions. Examples. Parametrized surfaces. Curves in surfaces and Tangent plane. Charts, coordinate vector fields, change of charts.
- **Topic 4.** Differentiable functions and maps. First fundamental form: lengths, angles and areas. Orientations.
- **Topic 5.** Geodesic and normal curvature. Second fundamental form and Gauss map. Types of points, principal, normal and Gauss curvature. Principal directions, asymptotic curves, umbilic points. Vector and direction fields
- **Topic 6.** Intrinsic Geometry. Covariant derivative and Gauss Theorema Egregium. Isometries, conformal maps and isothermal coordinates. Geodesics and exponential map: distance and convexity. Gauss-Bonnet Theorems.

Some other topics, as those related with global geometry of curves and surfaces will be developed by the students in groups: four-vertex theorem, regular neighbourhoods of compact curves and surfaces, differentiable Jordan curve theorem, Fenchel's theorem, hyperbolic geometry, minimal and ruled surfaces, etc.

4.4. Course planning and calendar

Classes are taught according to the academic calendar established by the University of Zaragoza and a schedule approved by the School of Sciences (see website): classes (lectures and problem sessions) meet three periods per week during the first semester and four periods per week in the second semester. Any key date for the classes (including schedules of labs) will be announced in advance on the Moodle platform.

Dates for the written test at the end of the first semester and at the end of the course are in accordance with the enabled period for exams within the academic calendar posted by the School of Science. The lab exam will be held in the last lab session.

4.5. Bibliography and recommended resources

- Do Carmo, Manfredo P., *Differential geometry of curves and surfaces*, Prentice-Hall, Inc., Englewood Cliffs, N.J}, 1976, viii+503.
- Cordero, Luis A. *Geometría diferencial de curvas y superficies con Mathematica* / Luis A. Cordero, Marisa Fernández, Alfred Gray . Buenos Aires. Addison-Wesley Iberoamericana, cop. 1995.
- Costa, Antonio F. *Notas de geometría diferencial de curvas y superficies* / Antonio F. Costa, Manuel Gamboa, Ana M. Porto Madrid : Sanz y Torres, D.L. 1997.
- Paul Zimmermann, Alexandre Casamayou, Nathann Cohen, Guillaume Connan, Thierry Dumont, Laurent Fousse, François Maltey, Matthias Meulien, Marc Mezzarobba, Clément Pernet, Nicolas M. Thiéry, Erik Bray, John Cremona, Marcelo Forets, Alexandru Ghitza, Hugh Thomas. Computational Mathematics with Sagemath. Disponible en https://www.sagemath.org/sagebook/english.html