

Academic Year/course: 2022/23

## 27009 - Ordinary Differential Equations

### Syllabus Information

**Academic Year:** 2022/23

**Subject:** 27009 - Ordinary Differential Equations

**Faculty / School:** 100 - Facultad de Ciencias

**Degree:** 453 - Degree in Mathematics

**ECTS:** 9.0

**Year:** 2

**Semester:** Annual

**Subject Type:** Compulsory

**Module:**

## 1. General information

### 1.1. Aims of the course

These approaches and objectives are aligned with the following Sustainable Development Goals (SDGs) of the United Nations 2030 Agenda (<https://www.un.org/sustainabledevelopment/es/>), in such a way that the acquisition of the learning outcomes of the module provides training and competence to contribute to some extent to their achievement: (4) Quality education, (5) Gender equality, (8) Decent work and economic growth, (9) Industry, innovation and infrastructure, (10) Reducing inequality, (17) Partnerships for the goals.

## 2. Learning goals

## 3. Assessment (1st and 2nd call)

## 4. Methodology, learning tasks, syllabus and resources

### 4.1. Methodological overview

The methodology followed in this course is oriented towards the achievement of the learning objectives. A wide range of teaching and learning tasks are implemented, such as lectures, problem-solving sessions and tutorials.

### 4.2. Learning tasks

This course is organized as follows:

- **Lectures.** Two weekly sessions. The teacher will provide explanations about the theory and abundant examples.
- **Problem-solving sessions.** One weekly session. Exercises will be solved by the student and presented to the group. Students will be required to gather in groups and to study a concrete problem and prepare a written report, to be discussed in the classroom.
- **Tutorials.** Students will be attended by the teacher at office hours.

The teaching activities and assessment tasks will take place in a face-to-face mode, except in the case that, due to the health situation, the dispositions emitted by the competent authorities and by the University of Zaragoza compel to take them to a greater or lesser extent in a telematic form.

### 4.3. Syllabus

**Section I.- Linear systems: constant coefficients**

- **Topic 1. Linear differential equations with constant coefficients**
  - First-order homogeneous equation
  - First-order nonhomogeneous equation
- **Topic 2. Linear systems: introduction**
  - Terminology and first properties
  - Eigenvectors and eigensolutions
  - Generalized eigenvectors and generalized eigensolutions
- **Topic 3. Exponential matrix**
  - Convergence
  - Exponential matrix definition and first properties
  - Exponential matrix via generalized eigensolutions
  - Differential of the exponential matrix
- **Topic 4. Linear systems**
  - Solution of homogeneous system
  - Solution of a nonhomogeneous system
  - Higher-order differential equations
- **Topic 5. Higher-order differential equations**
  - Solution of the homogeneous equation
  - Solution of the non homogeneous equation
  - Undetermined coefficients
- **Topic 6. Qualitative theory**
  - Notion of stability
  - Stability and spectrum
  - Phase portrait. Classification of 2-d systems.
- **Topic 7. Laplace transform**
  - Laplace transform defined
  - Calculus of Laplace transform
  - Calculus of inverse Laplace transform
  - Solution of initial value problems
  - Stability

## Section II.- Linear systems: general case

- **Topic 1. Linear equations**
  - Homogeneous equations
  - Nonhomogeneous equations
  - Grönwall inequality
- **Topic 2. Linear systems**
  - Existence and uniqueness of solutions (homogeneous system)
  - Superposition principle. Resolvent matrix
  - Nonhomogeneous equations
  - Higher-order equations
  - Stability\*
- **Topic 3. Periodic systems\***
  - Periodic solutions
  - Structure of the solution
  - Stability and resonance

## Section III.- Nonlinear systems

- **Topic 1. Autonomous equations**
  - Some examples and properties
  - Existence and uniqueness. Asymptotes
  - Qualitative analysis
- **Topic 2. Nonautonomous equations**
  - Exact equations
  - Integrating factors

- Other methods (separable, homogeneous,...)
- **Topic 3. Existence and uniqueness**
  - Lipschitz functions
  - Existence and uniqueness: Picard theorem
  - Maximal solution
  - Global solution
- **Topic 4. Numerical methods**
  - Euler methods. Taylor method
  - Convergence
  - Runge-Kutta method
  - Multistep methods\*
- **Topic 5. Regularity of the general solution**
  - Continuous dependence
  - Smooth dependence.
  - The variational equation
  - Trivialization\*
- **Topic 6. Qualitative theory**
  - Autonomous systems
  - Stability of equilibria: linearization method
  - Stability of equilibria: Lyapunov functions\*
  - Phase diagram

Topics marked with \* will be included only if time allows.

#### 4.4. Course planning and calendar

Further information concerning the timetable, classroom, office hours, assessment dates and other details regarding this course will be provided on the first day of class or please refer to the website of the Faculty of Sciences.

#### 4.5. Bibliography and recommended resources

- Simmons, George F.. Ecuaciones diferenciales : con aplicaciones y notas históricas / George F. Simmons ; con un capítulo sobre métodos numéricos de John S. Robertson ; traducción Lorenzo Abellanas Rapun . - 2a ed. Madrid [etc.] : McGraw-Hill, D.L. 2000.
- Boyce, William E.. Ecuaciones diferenciales y problemas con valores en la frontera / William E. Boyce, Richard C. DiPrima ; colaboración en la traducción Hugo Villagómez Velázquez . - 4a ed. México [etc.] : Limusa, cop.1998.
- Braun, Martin. Ecuaciones diferenciales y sus aplicaciones / M. Braun ; Traductor Ignacio Barradas Bribiesca . - [1a ed.] México : Grupo Editorial Iberoamérica, 1990.
- Hirsch, Morris W.. Ecuaciones diferenciales, sistemas dinámicos y álgebra lineal / Morris W. Hirsch, Stephen Smale ; versión española, Carlos Fernández Pérez Madrid : Alianza, 1983.
- Guzmán, Miguel de. Ecuaciones diferenciales ordinarias : teoría de estabilidad y control / M. de Guzmán . - [1a ed., reimp.] Madrid : Alhambra, 1987.
- Calvo Pinilla, M.. Curso de ecuaciones diferenciales ordinarias / Manuel Calvo Pinilla y Jesús Carnicer Álvarez Zaragoza : Prensas Universitarias de Zaragoza, 2010.
- Zill, Dennis G.. Ecuaciones diferenciales con aplicaciones de modelado / Dennis G. Zill . - 6a ed. México [etc.] : International Thomson Editores, cop. 1997.
- Marcellan, Francisco. Ecuaciones diferenciales : problemas lineales y aplicaciones / Francisco Marcellan, Luis Casaus, Alejandro Zarzo . - 1ª ed. en español, [reimp.] Madrid [etc.] : McGraw-Hill, D. L. 1991.

<http://psfunizar10.unizar.es/br13/egAsignaturas.php?codigo=27009>