Universidad Zaragoza

Academic Year/course: 2022/23

26907 - Algebra II

Syllabus Information

Academic Year: 2022/23
Subject: 26907 - Algebra II
Faculty / School: 100 - Facultad de Ciencias
Degree: 447 - Degree in Physics
ECTS: 6.0
Year: 1
Semester: Second semester
Subject Type: Basic Education
Module:

1. General information

1.1. Aims of the course

Rationale and goals:

The course aims at providing the students with a set of fundamental tools and results in Linear Algebra that will be applied to characterize the description of states and of operators in physical systems, as well as of the transformations representing changes in the reference system.

Together with "Algebra I", "Mathematical Analysis and Differential calculus" (in the first year) and "Geometry and Integral calculus", "Differential Equations", "Mathematical Methods in Physics" and "Computational Physics" (in later years), "Algebra II" provides the student with the necessary mathematical tools to study dynamical models and their solutions.

Despite this bias towards physical applications, it is worth mentioning that the mathematical tools provided are routinely applied also in several other fields, as chemistry, biology, computer science, economics, geology, among others.

Objectives of the course:

O1. Understanding the meaning of linear transformations and their matrix representation, as well as the importance of the choice of the basis. Understanding the importance of eigenvalue and eigenvectors, as well as the methods to find them out.

O2. Mastering the concepts of canonical forms of operators and operator functions.

O3. Understanding the meaning and role of scalar products, and as well as the concepts of orthogonality, orthonormal bases and orthonormalization.

O4. Understanding the importance of the transformations that preserve scalar products.

These objectives are not aligned to any of the Sustainable Development Goals of the UN 2030 Agenda (https://www.un.org/sustainabledevelopment/).

1.2. Context and importance of this course in the degree

The modeling of physical systems often involves their description in terms of vector spaces and of linear transformations defined on such spaces. It is therefore essential to know how to calculate the intrinsic algebraic features of a system, such as the set of all possible eigenvalues of a quantum operator, and to know which properties the reference systems, used in their description, have to fulfill.

1.3. Recommendations to take this course

Students are expected to attend Algebra I before this course.

2. Learning goals

2.1. Competences

1: Ability to calculate eigenvalues and eigenvectors of matrices and operators both analytically and numerically

2: Capacity to determine the canonical form of an operator and to use it to define operator functions

3: Ability to build orthonormal bases and to determine the components of a vector in such bases

4: knowledge of the properties of the eigenvalues and eigenvectors of relevant physical operators (projectors, self-adjoint operators, Hermitian operators, symmetric operators, orthogonal operators, ...)

5: Knowledge of the invariance groups of different scalar products (complex, Euclidean, Minkowski) both in its finite and infinitesimal versions.

2.2. Learning goals

In order to pass, the student should demonstrate the following skills:

1: he/she is able to perform simple matrix operations using numerical tools

2: he/she can determine the characteristic polynomial and generalized eigenspaces of an operator

3: he/she can calculate the exponential function of an operator. Example: application to the solution of harmonic oscillator.

4: he/she is able to orthonormalize a given basis, by the Gram-Schmidt procedure.

5: he/she knows how to relate unitary and orthogonal transformations to hermitian/symmetric operators, via exponentiation.

2.3. Importance of learning goals

The subject of Algebra II is of fundamental importance for the understanding of the tools used to solve classical dynamical systems, and absolutely necessary for the understanding of the basic concepts in quantum mechanics, that are always modeled using the concepts presented herein (or their generalizations to infinite dimensions).

3. Assessment (1st and 2nd call)

3.1. Assessment tasks (description of tasks, marking system and assessment criteria)

Students will prove their understanding and achievements through the following assessment methods:

1) (15% of the final grade). Continuous evaluation of the student's progress by direct interaction in the classroom, rewarding active participation during the lectures, and solution of the problems proposed by the teacher. On top of that, two on-line tests on parts of the program will be proposed.

2) (15% of the final grade). Evaluation of the skills acquired during the computer practice sessions. The results obtained during the practice sessions and/or a specific final exam will be considered for the final grade.

3) (70% of the final grade). A final exam on theory and problems. Depending on the circumstances, and if the students agree, a partial exam on theory and problems on the first chapters will be organized, so that students who reach 5 points out of 10 in the mid-term exam can skip the questions on the first chapters in the final exam.

It is also possible, for students that cannot attend the lectures on a regular basis, to pass the course through two global tests covering points 2,3 above. In that case, point 3 will be scored 85% of the final grade.

It will be necessary to reach a score of 4 out of 10 in both point 2 and 3 above and a global score of at least 5 points out of 10, to pass the course.

4. Methodology, learning tasks, syllabus and resources

4.1. Methodological overview

The learning process for this course is based on the following:

The lectures (theory session) must provide the student with the backbone of concepts and contents that will be further expanded through problems and practicas sessions. The optional home-work essay can provide more in-depth understanding on specific topics, that may be interesting to only a few students. The exercises should also serve as a self-assessment mechanism for the student and that is why participation in problem classes becomes a very important tool.

Course material: There are notes written by the teachers and available from the course's Moodle page.

All information on the course is presented on the first day of lectures, and is permanently available in the moodle page.

4.2. Learning tasks

Outline:

Teaching is structured in 4 hours of lecture per week (3 hours of theory + 1 hour of exercises and problems).

Likewise, the course includes 4 practice sessions of 2 hours each of computer practice during the semester, plus an introductory session of 2 hours.

In detail, teaching activities will include:

- Lectures, providing the theorems and demonstrations, organized in their logical layout according to the development of the program.

- Exercise sessions, to apply and consolidate the theoretical understanding by means of relevant examples and problems.

- Computer programming of problems in linear algebra, extending the scope of the classroom exercises to the cases where computations become too heavy (high dimensional vector spaces).

- Self-evaluation tests (optional) in the online-platform Moodle, allowing the students to assess their degree of understanding of the different topics.

4.3. Syllabus

1. Complex Vector Spaces and their endomorphisms

2: Multilinear mappings

3: Canonical forms of endomorphisms

4: Functions of operators

5: Vector spaces with scalar product

6: Endomorphisms on vector spaces with scalar product

4.4. Course planning and calendar

Lectures (Theory+Problems) will be held in the period February-May, according to the official calendar and schedule published in https://ciencias.unizar.es/calendario-y-horarios. Computer practice sessions will take place during the semester.

4.5. Bibliography and recommended resources

http://psfunizar10.unizar.es/br13/egAsignaturas.php?codigo=26907