Universidad Zaragoza

Academic Year/course: 2021/22

30605 - Mathematics II

Syllabus Information

Academic Year: 2021/22
Subject: 30605 - Mathematics II
Faculty / School: 109 - Facultad de Economía y Empresa
Degree: 432 - Joint Law - Business Administration and Management Programme
ECTS: 6.0
Year: 1
Semester: Second semester
Subject Type: Basic Education
Module:

1. General information

1.1. Aims of the course

The expected results of the course respond to the following general aims

The general objectives of the mathematical subjects in this Degree are included in the following two main goals: (1) Mathematical education, (2) Training to apply Mathematics to the challenges that the students will encounter in their careers.

The subject Mathematics II supposes a step forward in these objectives which Mathematics I also dealt with. Education in Mathematics is significant not only because of the transmission of new concepts, but also because the students gain a rigorous and accurate perspective, as well as the capacity for abstraction and the scientific method that characterise Mathematics. Regarding the second goal, this subject introduces students to modelling, using the mathematical analysis approach through two different ways: classical optimisation, and dynamical analysis.

These objectives are aligned with the Sustainable Development Objectives (ODS) of the Agenda 2030 and specific goals (https://www.un.org/sustainabledevelopment/es/), contributing to their achievement:

Objectives 1-17 (in all of them mathematical modelizations can be formulated).

1.2. Context and importance of this course in the degree

Mathematics II is a course of basic training of 6 ECTS that is taught in the second semester of the first academic year and which is the continuation of Mathematics I taught in the first semester of the same course, on whose concepts are based.

The course mathematics II is divided into two clearly distinct blocks: mathematical programming and dynamic analysis, which respond to two different points of view of economic reality. After the first, the student will know how to pose and solve a wide range of classic optimization problems: linear or nonlinear, without restrictions or with equality constraints. In the case of optimization programs in which both the objective function and the constraints are linear is used as a technique for resolution simplex method. This theme can be used to connect the traditional teaching of resolution with the use of computer programs that simplify the process of calculation and place the student in professional practice.

In the second block, dynamic analysis, it is solved differential equations and analyzed the solution. Its inclusion in the program is required because in the economic analysis it is usual that economic processes are not static, as for example: optimal economic growth, optimal management of renewable and non-renewable resources, optimum investment in the long term, etc.

As Mathematics is a tool and a support for other subjects that are essential in the education of the students (Microeconomics, Macroeconomics, Econometrics, etc.), Mathematics II continues the line of work of Mathematics I by bringing Mathematics closer to problems in economic scenarios, which will undoubtedly facilitate a deeper comprehension of and, as a consequence, better skills in applying Mathematics.

After passing the mathematical subjects in the Degree, the students will have worked towards attaining one of the most important goals of mathematical theory: to formulate models that explain the real world. Prospective graduates will be able to use the language of science and to understand the role played by Mathematics in the development of their thinking skills, given that the students' logical reasoning, accuracy, rigor, capacity for abstraction and skills in interpreting results will be improved. This is why the subjects of Mathematics are indispensable tools which allow the designing of appropriate models that are used for researching, describing, understanding and thinking about the realities of companies

1.3. Recommendations to take this course

The students should have a good command of all the contents of the subject Mathematics I, taught during the first semester of the first year. They must, in any case, know the meaning and implications of the differentiability of a function and be skilled in the calculus of partial derivatives. The students also have to know how to determine the sign of a quadratic form. They must also be able to present and support an argument with a logical sequence and to connect various mathematical aspects previously learnt.

2. Learning goals

2.1. Competences

After completing the course, the student will be competent in the following skills:

Problem solving

Analysis and synthesis

Decision-making.

Applying knowledge to practice

2.2. Learning goals

The student, in order to pass the course, will have to show her/his competence in the following skills:

1. To have gained good skills in using mathematical language, both in comprehension and writing.
2. To be able to identify the fundamental elements of an optimisation problem: variables, objective function and constraints.
3. To be able to formulate static optimisation problems: unconstrained, and with equality and/or inequality constraints.
4. To know how to solve an optimisation problem by the graphical method, when that is possible.
5. To be able to evaluate whether or not a mathematical programme meets the conditions that allow it to be solved by the techniques learnt.
6. To be able to distinguish between critical points and extrema (optima).
7. To be able to discriminate between local and global optima.
8. To be able to distinguish between necessary conditions and sufficient conditions for local optimality.
9. To be able to calculate the critical points by solving the system of equations obtained by applying the first-order conditions for local optimality, both for unconstrained cases and for problems with equality constraints.
10. To know how to classify the obtained critical points by using the second-order conditions, both for unconstrained optimisation programmes and for problems with equality constraints.
11. To be able to apply the conditions which guarantee that an optimum is global.
12. To be able to interpret economically the Lagrange multipliers obtained in an optimisation problem with equality constraints.
13. To be able to evaluate whether a mathematical programme is linear. If it is, they must know how to solve it by the graphical method (when that is possible) and by the simplex algorithm.
14. When varying a parameter of a linear optimisation programme, the students must be able to analyse how the solution changes, without solving the new problem.
15. To be able to use some computer programmes to find the solution to an optimisation problem and to be able to interpret the results obtained.
16. To be able to identify a dynamic process in an economic scenario and be able to represent this process (when possible) by an ordinary differential equation.
17. To understand the concept of the solution of an ordinary differential equation and to be able to distinguish between general solution and particular solution.
18. To be able to discriminate between a first-order differential equation and a linear differential equation of order n.
19. To be able to identify whether a first-order differential equation is with separable variables, homogeneous, exact, or of linear type, and to know how to solve the equation by the appropriate method.
20. For a linear differential equation with constant coefficients, they must be able to write the complementary (homogeneous) equation and obtain its general solution.
21. To be able to find a particular solution of a linear differential equation with constant coefficients.
22. To have the know-how to calculate the general solution of a linear differential equation with constant coefficients.
23. To be able to work out the solution of a linear differential equation of order n with constant coefficients, given n initial

conditions.

2.3. Importance of learning goals

They permit the comprehension of theoretical concepts and models that are part of the contents of other related subjects studied in the Degree. Mathematics is most important in this goal because it facilitates the analysis and discussion of the models and concepts studied. In this regard, it is worth mentioning that Optimisation techniques allow the laying of the foundations of the two basic paradigms of Microeconomics, namely, the theory of consumer choice and the production theory. The concepts of convex set and concave/convex function, whose economic interpretations are, respectively, the diversity in consumption and the law of diminishing marginal returns, have important applications. Linear Programming is very useful in production planning problems and it allows the solving of some simple exercises of comparative statics. Different techniques are required for the analysis of dynamic processes in continuous time, which is essential, for example, in models of economic growth. The theory of differential equations provides the necessary tools to deal with some key concepts such as trajectory over time, evolution of the system, stability, etc.

3. Assessment (1st and 2nd call)

3.1. Assessment tasks (description of tasks, marking system and assessment criteria)

The evaluation will be **GLOBAL** in the first and second sittings, and it will consits of a written final exam to be done on the dates established by the Faculty for that purpose.

These exams will be written and will assess the proposed learning outcomes by means of theoretical, practical and theoretic-practical questions that will fit the matter taught. Both they will be worth 10 points

In addition, in the first sitting, it will be possible for students to take a voluntary intermediate test worth 5 points. This test will assess the knowledge of the students about the topics of the first three Chapters of the program of the subject. This test will take place in the middle of the term, and the date will be announced in advance in class and/or the virtual teaching platform.

The students that obtain in the intermediate test a mark equal or greater than 50% of the maximum mark (2.5 out of 5) could eliminate the topics of the Chapters 1,2,3 out of the final exam in the first sitting. In that case, the corresponding mark of the eliminated topics will be incorporated to the mark of the global final exam with the weight that the said contents be considered in the final exam.

In order to pass the subject, students have to obtain at least 5 points out of 10 in any of the final marks. If the student obtains a mark of at least 2.5 points out of 5 in the intermediate test and wants to do the whole global exam anyway, the best of the two marks in the first part of the subject will be considered to compute the final mark.

To be eligible for this form of assessment students are required to participate actively and resolve issues, exercises and tests to be carried out in the classroom, according to indications that the teacher in charge of each group of the subject will be exhibiting the same day of the presentation. In such a case, it is necessary to attend and participate in at least 75% of the face-to-face sessions or proposed activities. The student that does not fulfill this requirement at the end of semester will not be subject to continue with this procedure of assessment.

It has to be taken into account that the evaluation process closes at the end of the academic year, so it is not possible to claim academic merits from one academic year in a later one.

Students taking their exams at the fifth or sixth opportunity will be marked following the rules established under the Governing Council Agreement on 22 December 2010, which sets out the assessment regulations in the University of Zaragoza.

It is expected that these tests will be done in person, but if the sanitary circumstances require it, they will be done in a semi-face or online. In the case of online evaluation, it is important to note that, in any test, the student can be recorded, being able to exercise their rights by the procedure indicated in: https://protecciondatos.unizar.es/sites/protecciondatos.unizar.es/files/users/lopd/gdocencia_reducida.pdf The necessary software will be used to check the originality of the activities carried out. The detection of plagiarism, copying or any other irregular practice in an activity will involve the rating of 0 points in it.

Evaluation Criteria

Students will be assessed on whether they have acquired the learning outcomes mentioned above. In particular, they will be assessed on the following aspects:

1. Correct mathematical writing.
2. Logical reasoning in the posing and solving of the problems.
3. Reference to the theoretical results used, when relevant.
4. The choice of the most appropriate method for the solving of problems.
5. Clarity in the application of mathematical concepts and procedures.
6. Computations carried out with care.
7. The correct expression of the results obtained when solving problems.

4. Methodology, learning tasks, syllabus and resources

4.1. Methodological overview

The learning process that has been designed for this course is based on the following activities:

The objective of this subject is that the students should develop the analytical skills, rigour and intuition needed for using mathematical concepts and results and that they should be able to apply these abilities to the analysis of problems of an economic nature. Therefore, the teaching should aim to provide students with a solid mathematical knowledge and to train them in a way of reasoning that will allow them thereafter to successfully solve a wide variety of questions in an economic scenario.

4.2. Learning tasks

Lectures (1.2. ECTS): 30 hours. At the same time, some exercises will be solved with the participation of the students to help them understand the theoretical concepts presented. These classes are face-to-face and will be given to the whole group.

Practical sessions (1.2 ECTS): 30 hours each subgroup. The students will apply the theoretical contents in order to solve, with the teacher's help, more complex exercises, and problems of an economic nature. Problem sheets will be available for the students and the teacher will announce in advance the problems that will be solved in each practical lesson so that the students can prepare them beforehand. These classes are face-to-face and will be given separately to each subgroup.

Seminars (practical sessions P6. 0.6 ECTS): 15 hours. It may consist of a number of different activities designed to support the learning process, including: follow-up of some simple projects that had been assigned to small teams of students and the presentation of these projects; answering questions that students may have regarding some of the contents taught; solving problems of an economic nature by using some of the mathematical tools taught during the classes, etc. These seminars may also be devoted to the teaching of more advanced topics, intended for the students interested in learning some further mathematical tools that would allow them to deal with more general problems. In this way, the students are shown that both Mathematics and Economics are vibrant sciences with many facets to be studied.

Both the splitting in two of the group in the practical sessions and the P6 activities will be subject to the availability of professors.

Autonomous work (3.6 ECTS): 75 hours.

The teaching methodology is expected to be face-to-face. However, if for public health reasons it were necessary, the classes might take place online

4.3. Syllabus

The contents detailed in the programme below will be developed in thelectures and practical sessions. Any variations in the order will be indicated by the teacher in the presentation of the course.

Chapter 1: Mathematical programs

1.1. General formulation of a mathematical program. Classification.

1.2. Definitions and properties. Weierstrass' Theorem.

1.3. Graphical solving.

1.4. Introduction to convexity:

1.4.1. Convex sets. Definition and properties.

1.4.2. Convex and concave functions. Definitions and properties.

1.4.3. Convex programs.

Chapter 2: Programming without constraints

2.1. Problem's formulation.

2.2. Local optima:

2.2.1. First order conditions for the existence of a local optimum.

2.2.2. Second order conditions for the existence of a local optimum.

2.3. Global optima: convex programs.

Chapter 3: Programming with equality constraints

3.1. Problem's formulation.

3.2. Local optima:

3.2.1. First order conditions for the existence of a local optimum.

3.2.2. Second order conditions for the existence of a local optimum.
3.3. Global optima: convex programs and Weierstrass' Theorem.
3.4. Economic interpretation of the Lagrange's multipliers.

Chapter 4: Linear programming

4.1. Formulation of a problem of linear programming.
4.2. Solutions of a linear program. Basic feasible solutions.
4.3. Characterization of the optimal basic feasible solutions. Simplex' Algorithm.
4.4. Introduction to the sensitivity analysis.
4.5. Introduction to the dual program.

Chapter 5: Introduction to ordinary differential equations
5.1. Introduction to the dynamical analysis.
5.2. Concept of differential equation, solution and types of solution.
5.3. First order ordinary differential equations:
5.3.1. Separable equations.
5.3.2. Linear first order equations.
5.4. Linear differential equations of order n with constant coefficients.
5.5. Qualitative analysis: equilibrium points and stability.

4.4. Course planning and calendar

1. Presentation of the subject in the first session of the semester, in accordance with the timetable established by the Faculty.
2. Continual attendance at, and productive use of, theoretical and practical classes.
3. Attendance at practical classes P6, which may include computer practice if the global schedule allows it.
4. Midterm exams, scheduled in accordance with the academic calendar.
5. Final exam, on the day established by the Faculty.