Universidad Zaragoza

Academic Year/course: 2021/22

27032 - Probability Theory

Syllabus Information

Academic Year: 2021/22
Subject: 27032 - Probability Theory
Faculty / School: 100 - Facultad de Ciencias
Degree: 453 - Degree in Mathematics
ECTS: 6.0
Year: 4
Semester: Second semester
Subject Type: Optional
Module:

1. General information

2. Learning goals

3. Assessment (1st and 2nd call)

4. Methodology, learning tasks, syllabus and resources

4.1. Methodological overview

The methodology followed in this course is oriented towards the achievement of the learning objectives. A wide range of teaching and learning tasks are implemented, such as lectures, practice sessions, seminars and tutorials.

4.2. Learning tasks

This course is organized as follows:

- **Lectures.** Two weekly hours. Theory will be illustrated by means of a variety of examples, trying to motivate the student?s participation. Interconnections with other mathematical subjects, coming from analysis and applied mathematics will also be discussed. Whenever appropriate, computer presentations will be used. Lecture notes, list of exercises, complementary material and references will be available for all of the students in the course.
- **Practice sessions.** Two weekly hours of practice sessions and seminars. In which different exercises and questions will be solved in detail. The students will have in advance the collection of such exercises, in order to facilitate their homework.
- **Seminars** in which the students will expose the work done. Discussions will be focused on the different ways to face problems, paying special attention to the writing from a mathematical point of view.
- **Tutorials.** Teachers will attend students individually in order to monitorize the practical work assigned to the them and to correct the ways of working.
- **Autonomous work and study.** Individually and in groups.

The teaching activities and assessment tasks will take place in a face-to-face mode, except in the case that, due to the health situation, the dispositions emitted by the competent authorities and by the University of Zaragoza compel to take them to a greater or lesser extent in a telematic form.

4.3. Syllabus

This course will address the following topics:

Section I. Probability spaces and random variables.

- **Topic 1. Probability and measure.**
  - Measure: events, sigma algebras of events and non measurable sets. Probability spaces: basic properties. Construction of probability measures.
- **Topic 2. Random variables.**
  - Random variables: definition and properties. Probability image and distribution function. Relation between distribution functions and random variables. Expectations: classical inequalities. Product spaces and random vectors. Conditional distributions: probability kernels and disintegration of probabilities. Independence of random variables. Borel-Cantelli lemmas.

Section II. Types of convergence and laws of large numbers.

- **Topic 3. Convergence of random variables.**
  - Types of convergence: in law, in probability, in mean of order p and almost surely. Relations and limit theorems.
- **Topic 4. Laws of large numbers.**
  - Weak laws of large numbers. Convergence of random series and strong laws of large numbers. Applications: normal numbers, Monte Carlo methods, probabilistic methods in approximation theory.

Section III. Central limit theorems.

- **Topic 5. Characteristic functions.**
  - Definition and basic properties. Derivatives and moments. Uniqueness, inversion and continuity theorems. Applications in the case of sums of independent random variables.
- **Topic 6. Central limit theorems.**
  - Historical review. Theorems of Moivre-Laplace, Lévy, Liapunov, and Lindeberg-Feller. Applications: construction of asymptotic confidence intervals. Rate of convergence in the central limit theorem: the Berry-Esseen theorem.

Section IV. Introduction to stochastic processes.

- **Topic 7. Markov chains.**
  - Definition and construction of a Markov chain. Examples: random walks, queues, and physical biological and economic models. The Chapman-Kolmogorov equation. Classification of states: periodicity, persistent and transient states. Stopping times: the strong Markov property. Limit distributions and mean recurrence times. Stationary distributions. Ergodic theorems. Applications in physical, biological, and economic models.
- **Topic 8. Introduction to stochastic processes in continuous time.**
  - The Kolmogorov consistency theorem. The Poisson process: construction and properties. Non homogeneous and compound Poisson processes: applications. Continuous time Markov chains: applications. Introduction to Brownian motion.

4.4. Course planning and calendar

Further information concerning the timetable, classroom, office hours, assessment dates and other details regarding this course will be provided on the first day of class or please refer to the Faculty of Sciences website and Moodle.

4.5. Bibliography and recommended resources

- Billingsley, Patrick. Probability and Measure / Patrick Billingsley . - 3rd ed. New York [etc.] : John Wiley, cop. 1995
- Çinlar, Erhan. Probability and Stochastics / Erhan Cinlar . New York : Springer, 2011.
- Grimmett, Geoffrey. Probability and Random Processes / Geoffrey Grimmett and David Stirzaker . - 3rd. ed., repr. with corr. Oxford : Oxford University Press, 2004.
- Gut, Allan. Probability: A Graduate Course. Springer. 2005.
- Norris, J.R.. Markov Chains. Cambridge University Press. 1997.
- Resnick, Sidney. Adventures in Stochastic Processes / Sidney Resnick Boston [etc] : Birkhäuser, cop.1992.
- Ross, Sheldon M.. Stochastic Processes / Sheldon M. Ross . - 2nd. ed. New York [etc.] : John Wiley and Sons, cop. 1996.
- Vélez Ibarrola, Ricardo. Cálculo de Probabilidades 2 / Ricardo Vélez Ibarrola . - [1ª ed.] Madrid : Ediciones Académicas, 2004.

http://psfunizar10.unizar.es/br13/egAsignaturas.php?codigo=27032