

## 27004 - Numbers and Sets

### Syllabus Information

**Academic Year:** 2021/22

**Subject:** 27004 - Números y conjuntos

**Faculty / School:** 100 - Facultad de Ciencias

**Degree:** 453 - Degree in Mathematics

**ECTS:** 6.0

**Year:** 1

**Semester:** First semester

**Subject Type:** Basic Education

**Module:**

## 1. General information

### 1.1. Aims of the course

The subject and its intended results correspond to the following approach and objectives: this is a subject which aims to lay a ground material and training for the degree. The main problem that the student faces at the beginning of the study of Mathematics is the acquisition of the language and general methods of mathematical expression, which to some extent amounts to laying the concepts that serve as a foundation of mathematical knowledge in the understanding that at this level they can only be an elementary version of these. The objective of the present course is providing such a foundation.

### 1.2. Context and importance of this course in the degree

This course lies within the module Algebraic Structures, although its transversal character makes the competences acquired therein necessary in the rest of the modules of the degree. In this course the stress is placed on the transversal competence C5T. To be able to obtain effective information from bibliographic and informatic resources, of great importance in all the subjects studied in the degree.

### 1.3. Recommendations to take this course

The attendance to the theoretical classes (presential or on line) and the personal work, together with the work on the proposed questions and problems, and the use of the the tutorage provided by the teacher.

## 2. Learning goals

### 2.1. Competences

The overcoming of the subject will allow the student to be more competent in the development of the objectives described in the section on the results of the learning exposed in that section (learning goals, see below). Moreover, the student will improve her/his performance in the following competences:

CT1, CT3, CE1, CE3, CT5 described in the legislation, and to be able of obtaining information by bibliographic, and informatic means.

### 2.2. Learning goals

To overcome the present subject, the student will be able to show the following skills:

- He/she makes proper use of the language and the basic properties of set theory, and its applications
- He/she makes proper use of natural numbers, the induction principle, and the basic combinatorial arguments
- He/she makes use of the basic results of the arithmetic of integers, and modular arithmetic.
- He/she has a basic knowledge of the arithmetic construction of the rational, and to some extent of the real numbers.

### 2.3. Importance of learning goals

The results of the learning of the course are very important since they provide a ground formation for the degree. In this course the student acquires competences on the use of mathematical language and the fundamental concepts of mathematics, whose lack prevents the adequate approach to the mathematical problems. That acquisition is also obtained in the courses of Linear Algebra, and Calculus

### 3. Assessment (1st and 2nd call)

#### 3.1. Assessment tasks (description of tasks, marking system and assessment criteria)

- Partial exam on Set Theory (eliminary for the final exam) 40% of the final exam.
- Final exam. If the part of Set Theory has been approved, it can only be presented to the rest, which in this case will be 60% of the grade.

In any case, the student will have the right to take the complete final exam and the final grade will be the best for the student, counting the partial exam or not.

Completion of the course "Basic Digital Competence: learn to inform yourself, create and communicate digitally (basic level)" and obtain the pass. At any rate the student has the right to base his/her final grade on just a global exam.

### 4. Methodology, learning tasks, syllabus and resources

#### 4.1. Methodological overview

The methodology followed in this course is oriented towards the achievement of the learning objectives. A wide range of teaching and learning tasks are implemented, such as lectures, problem-solving sessions, tutorials and autonomous work and study.

#### 4.2. Learning tasks

This course is organized as follows:

- **Lectures.**
- **Problem-solving sessions.** Participatory sessions addressing issues and problems.
- **Tutorials.**
- **Autonomous work and study.**

For the online course "Information Management" a training session of 50 minutes is expected, in order to explain to students the objectives and mechanics of operation of the virtual course in Moodle.

The teaching activities and assessment tasks will take place in a face-to-face mode, except in the case that, due to the health situation, the dispositions emitted by the competent authorities and by the University of Zaragoza compel to take them to a greater or lesser extent in a telematic form.

#### 4.3. Syllabus

This course will address the following topics:

- **Topic 0: Origins of number theory.**
  - Natural numbers.
  - Induction principle.
  - Basic arithmetics.
  - Number theory in Antiquity.
- **Topic 1: Sets.**
  - Basic notations.
  - Axioms: constructions and operations with sets.
  - Maps and relations.
  - Axiom of choice.
  - Notions on cardinals.
- **Topic 2: Natural numbers and integers.**
  - Peano Axioms. Natural numbers and set theory.
  - Operations and order relation in the set of natural numbers.
  - Construction of the set of integers from the set of natural numbers.
  - Euclidean algorithm, Bezout's identity and diophantine linear equations. Congruences.

- **Topic 3: Fields of numbers.**

- Rational numbers (construction from  $\mathbb{Z}$ , operations and order relation).
- Real numbers (approximation to their construction in set theory through Dedekind's cuts).
- Complex numbers (operations, geometric representation).

#### **4.4. Course planning and calendar**

Further information concerning the timetable, classroom, office hours, assessment dates and other details regarding this course will be provided on the first day of class or please refer to the Faculty of Sciences website and Moodle.

#### **4.5. Bibliography and recommended resources**

- Boyer, C. Historia de las Matemáticas, Alianza Editorial, Madrid, 1986
- Dedekind, R. ¿Qué son y para qué sirven los números. Alianza Editorial, Madrid 1998
- Ebbinghaus, H-D. et al. Numbers, Springer, New York, 1991
- Gerstein, L. J. Introduction to Mathematical Structures and Proofs, Springer, New York, 2012
- Halmos, P. R. Naive Set Theory, Van Nostrand, New York, 1960
- Kline, M. El pensamiento matemático de la antigüedad a nuestros días vols. I and III. Alianza Editorial, Madrid, 1992
- Tattersall, J. I. Elementary Number Theory, in Nine Chapters, Cambridge Univ. Press, 1999
- Stewart, I. and Tall. D. The foundations of Mathematics, Oxford University Press, 1977

<http://psfunizar10.unizar.es/br13/egAsignaturas.php?codigo=27004>