Universidad Zaragoza

Year : 2019/20

27035 - Fourier Analysis

Syllabus Information

Academic Year: 2019/20
Subject: 27035 - Fourier Analysis
Faculty / School: 100 -

Degree: 453 - Degree in Mathematics

ECTS: 6.0
Year: 4
Semester: Second semester
Subject Type: Optional
Module: ---

1.General information

1.1.Aims of the course

This is an optional course in the degree of Mathematics. Its goal is to present the essentials of Fourier analysis.

1.2.Context and importance of this course in the degree

The subject belongs to the module *Ampliación de análisis matemático*. To take this subject it is advisable to have passed the subjects of the module *Iniciación al análisis matemático*, that is, *Análisis matemático I*, *Análisis matemático II* and *Variable compleja*, as well as the subject *Integral de Lebesgue*. The subject *Análisis funcional* is a good companion.

1.3.Recommendations to take this course

The attendace to the class lectures and the computer laboratory sessions is advisable, as well as the individual work on the problems posed along the course and the use of the individual tutorization.

It is highly advisable to have passed the module *Iniciación al Análisis matemático*, which comprises the subjects *Análisis matemático I*, *Análisis matemático II* and *Variable compleja*. The subject requires a good knowledge of the Lebesgue integral and the Lebesgue spaces L^1 and L^2.

2.Learning goals

2.1.Competences

2.2.Learning goals

At the end of this course students should be able to:

- Know that a periodic function is determined by its Fourier coefficients and understand some convergence results of Fourier series.
- Know how the Fourier coefficients can be obtained by the discrete Fourier transform, and use the basics of the fast Fourier transform.
- Adapt the theory to non-periodic functions with the Fourier transform and understand some inversion results.

2.3.Importance of learning goals

3.Assessment (1st and 2nd call)

3.1.Assessment tasks (description of tasks, marking system and assessment criteria)

As a general rule, the module can be passed either showing a regular work along the academic year, or by a final exam.

- *Regular work.* During the course, the student results will be evaluated through a periodical supply of exercises or short tasks, together with their active participation during the course. The use of LaTeX in written presentations is

recommended; the evaluation can also include oral presentations. These evaluations will constitute the final mark.

- *Final exam.* The aforementioned procedure does not exclude the right, according to the current regulations, to a final exam which, by itself, allows to pass the module.

4.Methodology, learning tasks, syllabus and resources

4.1.Methodological overview

The methodology followed in this course is oriented towards the achievement of the learning objectives. A wide range of teaching and learning tasks are implemented, such as lectures, computer laboratory sessions, tutorials and autonomous work and study.

4.2.Learning tasks

This course is organized as follows:

- **Lectures**. Theoretical results will be explained.
- **Computer laboratory sessions** using software tools to illustrate applications of the theoretical results seen in lectures.
- **Tutorials**. Teachers will attend students individually.
- **Autonomous work and study**. Use of the computer tools of the University, mainly the computer laboratories and Moodle.
- **Assignments**. Problems or homework tasks will be posed during the course, allowing to pass the course.

4.3.Syllabus

This course will address the following topics:

- **Topic 1. Historical, physical and mathematical introduction.**
  - The vibrating string and the wave equation: D'Alembert, Euler, and Bernoulli. The heat transmission and its equation: Fourier. The concept of function: measure theory and functional analysis. The electromagnetic waves.
- **Topic 2. Preliminary mathematics.**
  - Banach spaces of continuous, differentiable, and integrable functions. Convergence of sequences and series of functions. Periodic functions, the torus and a little bit of complex analysis.
- **Topic 3. Fourier series.**
  - Formal Fourier sine, cosine and exponential series. Statement of the problem of the convergence of Fourier series: convolution, kernels, the unit circle, and its relation with complex analysis and the involved spaces. Pointwise, uniform and mean convergence results: summabilities of Fourier series. Riemann-Lebesgue lemma. Dirichlet's theorem and the Gibbs phenomenon. Riemann's localization principle. Exploiting the orthogonality: Hilbert spaces and Plancherel's theorem.
- **Topic 4. Discrete Fourier transform.**
  - Periodic sequences. The discrete transform and its inverse. Sampling and interpolation. Approximate calculus of Fourier coefficients. The FFT algorithm and its use in computer programs (Python).
- **Topic 5. Fourier transform.**
  - The continuous analog of Fourier series. Continuous frequencies. The Schwartz class of functions. Poisson and Gauss-Weierstrass kernels. The inversion formula. Fourier transform and L2 theory. Band limited functions. The uncertainty principle.

4.4.Course planning and calendar

As a general rule, this course has four face-to-face weekly hours. The schedule is set and made public by the Faculty of Science well before the beginning of the academic year.

Further information concerning the timetable, classroom, office hours, assessment dates and other details regarding this course will be provided on the first day of class or please refer to the Faculty of Sciences website and Moodle.

4.5.Bibliography and recommended resources

- F. J. Ruiz Blasco: *Análisis de Fourier* (in Spanish). Available for students in the *Anillo digital docente* of the University of Zaragoza.
- E. M. Stein and R. Shakarchi: *Fourier analysis. An introduction*. Princeton Lectures in Analysis, 1. Princeton University Press, Princeton, NJ, 2003. xvi+311 pp. ISBN: 0-691-11384-X.

- J. Duoandikoetxea: *Fourier analysis*. Translated and revised from the 1995 Spanish original by David Cruz-Uribe. Graduate Studies in Mathematics, 29. American Mathematical Society, Providence, RI, 2001. xviii+222 pp. ISBN: 0-8218-2172-5.
- Y. Katznelson: *An introduction to harmonic analysis*. Third edition. Cambridge Mathematical Library. Cambridge University Press, Cambridge, 2004. xviii+314 pp. ISBN: 0-521-83829-0; 0-521-54359-2.
- W. Rudin: *Real and complex analysis*. Third edition. McGraw-Hill Book Co., New York, 1987. xiv+416 pp. ISBN: 0-07-054234-1.